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Linear relations amongst sums of two squares. (English) Zbl 1161.11387

Reid, Miles (ed.) et al., Number theory and algebraic geometry. To Peter Swinnerton-Dyer on his 75th birthday. Cambridge: Cambridge University Press (ISBN 0-521-54518-8/pbk). London Mathematical Society Lecture Note Series 303, 133-176 (2003).

Consider the sums of two squares. It is easy to show that this set contains infinitely many arithmetic progressions of 4 distinct integers. The question of frequency of such progressions is addressed.

Let $r(n)$ denote the number of representations of n as a sum of two squares. Then the author shows that

$$\sum_{a<b<c<d\leq X} r(a)r(b)r(c)r(d) = CX^2 + E(X),$$

where a, b, c, d are restricted to arithmetic progressions of length 4, $E(X) \ll X^2 \log^{\delta} X$ for some explicitly given $\delta > 1/25$, and $C > 0$ is a constant.

The result is also related to the problem of counting rational points on varieties of the type $L_1(x_1, x_2)L_2(x_1, x_2) = x_3^2 + x_4^2$, $L_3(x_1, x_2)L_4(x_1, x_2) = x_5^2 + x_6^2$.

Moreover estimates of the sum

$$\sum_{x\in\mathcal{R}_2} r\big(L_1(\mathbf{x})L_2(\mathbf{x})\big)r\big(L_3(\mathbf{x})L_4(\mathbf{x})\big)$$

are proved. Several interesting choices of the forms L_i are investigated in more detail.

For the entire collection see [Zbl 1050.11004].

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MSC:

11N25	Distribution of integers with specified multiplicative constraints	Cited in **6** Reviews
11E25	Sums of squares and representations by other particular quadratic forms	Cited in **4** Documents

Keywords:

arithmetic progressions

Full Text: Link