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Fourier series and integral equation method for the exterior Stokes problem. (English)

Zbl 1161.65087

Numer. Methods Partial Differ. Equations 24, No. 3, 699-727 (2008).

One of the objectives of this paper is to get more insights on the theoretical and constructive treatments of edge singularities of the three-dimensional Stokes system. The author investigates the following Stokes system of partial differential equations

$$\begin{aligned} -\nu\Delta U + \nabla p &= 0 & \text{in } Q', \\ \nabla \cdot U &= 0 & \text{in } Q', \end{aligned}$$

with the boundary condition

$$U = F \text{ on } \Gamma \times [0, 2\pi],$$

and the periodic condition

$$U(\cdot, 0) = U(\cdot, 2\pi) \quad \text{on } \mathbb{R}^2.$$

Here the unbounded domain $Q' := \Omega' \times (0, 2\pi)$ is the exterior to the prism $Q := \Omega \times (0, 2\pi)$, where $\Omega := \mathbb{R}^2 \setminus \overline{\Omega'}$ is a bounded simply connected domain with polygonal boundary $\Gamma \equiv \partial\Omega$.

First, the author proves the well-posedness of the coupled exterior-interior problem for the Stokes operator in suitably defined weighted Sobolev spaces. Second, by means of Fourier series in the z -variable, the problem is reduced to finding Fourier coefficients via boundary integral equations of hydrodynamic potential theory. The global regularity of the solutions of the integral equations is investigated in appropriate weighted Sobolev spaces of traces. Finally, the last part of the paper is devoted to an optimal convergent boundary element method for the integral equations. This provides optimal convergent semi- and fully-discrete spectral methods of Fourier-Galerkin type.

Reviewer: [Petr Necesal \(Plzen\)](#)

MSC:

- 65N38 Boundary element methods for boundary value problems involving PDEs
- 65N12 Stability and convergence of numerical methods for boundary value problems involving PDEs
- 35Q30 Navier-Stokes equations
- 76D07 Stokes and related (Oseen, etc.) flows
- 76M15 Boundary element methods applied to problems in fluid mechanics
- 35J25 Boundary value problems for second-order elliptic equations
- 65N15 Error bounds for boundary value problems involving PDEs

Keywords:

boundary element; error estimates; Fourier series; refined mesh; regularity; convergence; edge singularities; Stokes system; exterior-interior problem; boundary integral equations; boundary element method; spectral methods

Full Text: [DOI](#)

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