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**FGMRES preconditioning by symmetric/skew-symmetric decomposition of generalized Stokes problems.** (English) [Zbl 1161.76033](#)

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**Summary:** The generalized Stokes problem is solved for non-standard boundary conditions. This problem arises after time semi-discretization by ALE method of the Navier-Stokes system, which describes the flow of two immiscible fluids with similar densities but different viscosities in a horizontal pipe, when modeling heavy crude oil transportation. We discretized the generalized Stokes problem in space using the “mini”-finite element. The inf-sup condition is proved when the interface between the two fluids and its discretization match exactly. The linear system obtained after discretization is solved using different iterative Krylov methods with and without preconditioning. Numerical experiments with different meshes are presented as well as comparisons between the methods considered. The results suggest that FGMRES and a preconditioning technique based on symmetric/skew-symmetric decomposition is a promising candidate for solving large scale generalized Stokes problem.

**MSC:**

[76M10](#) Finite element methods applied to problems in fluid mechanics

[76D05](#) Navier-Stokes equations for incompressible viscous fluids

[65F10](#) Iterative numerical methods for linear systems

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**Keywords:**

two-fluid flow; ALE method; mini-finite element; inf-sup condition; iterative Krylov methods

**Software:**

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