Robinson, James C.

**Linear embeddings of finite-dimensional subsets of Banach spaces into Euclidean spaces.**

(English) Zbl 1163.54008
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A Borel subset $S$ of a normed linear space $V$ is called *prevalent* if there exists a compactly supported probability measure $\mu$ such that $\mu(v+S)=1$ for all $v \in V$. This definition was introduced in a slightly different form in [*B. R. Hunt, T. D. Sauer* and *J. A. Yorke*, Bull. Am. Math. Soc. (N.S.) 27, No. 2, 217–238 (1992; Zbl 0763.28009)]. For separable spaces an equivalent definition was introduced earlier in [*J. P. R. Christensen*, Isr. J. Math. 13(1972), Proc. internat. Sympos. partial diff. Equ. Geometry normed lin. Spaces II, 255-260 (1973; Zbl 0249.43002)].

The main purpose of the paper is to study the prevalence of sets of linear maps with finite-dimensional ranges which are one-to-one (or even better) on a given compact subset $X$ of a Banach space under the assumption that $X$ is finite-dimensional in a certain metric sense.

Some of the main results:

(1) (Theorem 3.1) Let $X$ be a compact subset of a Banach space $B$ such that the Hausdorff dimension of $X-X$ is $< k$, where $k$ is a positive integer. Then a prevalent set of linear maps $L : B \to \mathbb{R}^k$ are one-to-one between $X$ and its image.

(2) (Theorem 5.1) For the upper box-counting dimension $d_B(X)$ the author proves that a prevalent set of such linear maps have Hölder continuous inverses on the image of $X$ if the box-counting dimension of $X$ is finite and $k > 2d_B(X)$.

(3) (Theorem 6.4) For the Assouad dimension $d_A$ the author of proves that if $k > d_A(X-X)$, a prevalent set of such linear maps have inverses of the image of $X$ which are Lipschitz up to a logarithmic term.

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**MSC:**

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|---|---|
| 54C25 | Embedding |
| 37C45 | Dimension theory of smooth dynamical systems |
| 46B20 | Geometry and structure of normed linear spaces |
| 54F45 | Dimension theory in general topology |



**Keywords:**

Assouad dimension; box-counting dimension; Hausdorff dimension; prevalent set

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