Alon, Noga; Lubetzky, Eyal
Privileged users in zero-error transmission over a noisy channel. (English) Zbl 1164.05033 \newline Combinatorica 27, No. 6, 737-743 (2007).

Summary: The $k$-th power of a graph $G$ is the graph whose vertex set is $V(G)^k$, where two distinct $k$-tuples are adjacent iff they are equal or adjacent in $G$ in each coordinate. The Shannon capacity of $G$, $c(G)$, is $\lim_{k \to \infty} \alpha(G^k)/k$, where $\alpha(G)$ denotes the independence number of $G$. When $G$ is the characteristic graph of a channel $C$, $c(G)$ measures the effective alphabet size of $C$ in a zero-error protocol. A sum of channels, $C = \sum_i C_i$, describes a setting when there are $t \geq 2$ senders, each with his own channel $C_i$, and each letter in a word can be selected from any of the channels. This corresponds to a disjoint union of the characteristic graphs, $G = \sum_i G_i$. It is well known that $c(G) \geq \sum_i c(G_i)$, and in N. Alon [Combinatorica 18, 301-310 (1998; Zbl 0921.05039)] it is shown that in fact $c(G)$ can be larger than any fixed power of the above sum.

We extend the ideas of [N. Alon, “Shannon capacity of a union,” Combinatorica 18, No. 3, 301–310 (1998; Zbl 0921.05039)] and show that for every $F$, a family of subsets of $[t]$, it is possible to assign a channel $C_i$ to each sender $i \in [t]$, such that the capacity of a group of senders $X \subseteq [t]$ is high iff $X$ contains some $F \in F$. This corresponds to a case where only privileged subsets of senders are allowed to transmit in a high rate. For instance, as an analogue to secret sharing, it is possible to ensure that whenever at least $k$ senders combine their channels, they obtain a high capacity, however every group of $k-1$ senders has a low capacity (and yet is not totally denied of service). In the process, we obtain an explicit Ramsey construction of an edge-coloring of the complete graph on $n$ vertices by $t$ colors, where every induced subgraph on $\exp(\Omega(\log n \log \log n))$ vertices contains all $t$ colors.

MSC:
- 05C35 Extremal problems in graph theory
- 05C55 Generalized Ramsey theory
- 94A24 Coding theorems (Shannon theory)

Keywords:
noisy channel; zero-error transmission; Shannon capacity; disjoint union of characteristic graphs; privileged subsets of senders; secret sharing; Ramsey construction

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References:

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