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Positive elements and automatic continuity of algebra morphisms. (English) Zbl 1164.46017

A locally convex algebra (l.c.a) is a locally convex space with a separately continuous multiplication. An l.m.c.a. is a locally convex algebra where the defining seminorms \( (p_\alpha) \) satisfy \( p_\alpha(xy) \leq p_\alpha(x)p_\alpha(y) \) for all \( x, y \) and \( \alpha \). A unital algebra \( A \) is a \( Q \) algebra if the set of its invertible elements is open in \( A \). An l.c.a. with a continuous involution is denoted by \( * \)-l.c.a.. The authors show that every \( * \)-morphism from a \( * \)-l.c.a \( A \) with continuous product into a Hermitian l.c.a. \( B \) is continuous whenever the set of positive elements of \( B \) is a normal cone. This latter condition is satisfied in every \( C^* \)-algebra. The authors also give a result on the uniqueness of the topology for Hermitian l.m.c.a with a specific cone of positive elements. They obtain also that every \( * \)-morphism from a \( Q * \)-l.c.a \( A \), with continuous inverse and product, into a commutative GB*-algebra is continuous. For the non-involutive case, they consider, in an l.c.a. \( B \), the set \( B^i_+ = \{ x \in B : Sp(x) \subset R_+ + iR_+ \} \). If \( B^i_+ \) is a normal cone, then every morphism from a \( Q \)-l.c.a. \( A \) into the algebra \( B \) is continuous. Consequences and applications are also given.

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46H40 Automatic continuity

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