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Stability of peakons for the Degasperis-Procesi equation. (English) Zbl 1165.35045

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The paper aims to prove the orbital stability of peakon solutions in the integrable Degasperis-Procesi (DP) equation,

$$y_t + y_x u + 3yu_x = 0,\ y \equiv u - u_{xx},$$

which resembles another integrable model, Camassa-Holm (CS) equation,

$$y_t + y_x u + 3yu_x = 0,\ y \equiv u - u_{xx},$$

although being essentially different from it. Nevertheless, the simplest peakon solutions are identical in both equations,

$$u(x,t) = c\exp(-|x-ct|),$$

with arbitrary positive constant c. Unlike peakons, smooth solutions to these equations feature an infinite propagation velocity. The DP equation also has exact solutions in the form of "shock peakons",

$$u(x,t) = -\operatorname{sgn}(x)\exp(-|x|)/(t+T), T > 0,$$

which can be generated by a collision of a symmetric pair of peakons, set initially at a finite distance. The stability of peakons within the framework of the CS equation was established previously. The difference in the proof of the peakon stability in the DP equation, which is the main result of this paper, is related to the different form of its dynamical invariants. In particular, the second invariant, $\int_{-\infty}^{+\infty} yvdx$, where $v(x)$ is defined by relation $(4-\partial_x^2)v = u$, includes the nonlocal density. The final result is that a slightly perturbed peakon evolves into a solution close to another peakon, which may have a different velocity.

Reviewer: Boris A. Malomed (Tel Aviv)

MSC:

35Q53	KdV equations (Korteweg-de Vries equations)
35B40	Asymptotic behavior of solutions to PDEs
35B35	Stability in context of PDEs
76B15	Water waves, gravity waves; dispersion and scattering, nonlinear interaction

Keywords:

water waves; Korteweg - de Vries equation; wave breaking

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