Schwenk-Schellschmidt, Angela

Pedal curves. I: Homogeneous differential equation. (English) Zbl 1165.53005

Result. Math. 52, No. 3-4, 369-382 (2008).

The author studies planar curves with the property that associated curves like the evolute, pedal curve, caustic curves, etc. are of the same shape as the original curve. The main result characterizes the support function of a logarithmic spiral by a differential equation and states that a regular curve c is a logarithmic spiral with center p if either the pedal curve with respect to p, the evolute, the co-pedal curve with respect to p (obtained by projecting the spiral center orthogonally onto the curve normals), or the caustic with respect to p are logarithmic spirals with center p. Further characterizations of the logarithmic spiral in terms of relations between the original curve and derived curves are provided as well.

The main tool in the proof of these equivalence statements consists in computing "quotient of derived curves", that is, functions of the spherical curve parameter that are obtained by complex division of the parametrized equations. For the curves in question, these quotients are constant.

The article also contains a few side results such as a generalization of an old theorem by *E. Weyr* [Schlömilch Z. XIV. 376–381 (1869; JFM 02.0422.01)] on a relation between a caustic and the evolute of the pedal curve, properties of the pedal, the evolute and the co-pedal operators on the space of spherically parametrized regular curves, and a characterization of ellipse and hyperbola as curves possessing a caustic that degenerates to a single point.

Reviewer: Hans-Peter Schröcker (Innsbruck)

MSC:

53A04 Curves in Euclidean and related spaces
34A30 Linear ordinary differential equations and systems

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Euclidean plane curves; curvature function; support function; pedal curve

Full Text: DOI