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Formal GNS construction and WKB expansion in deformation quantization. (English)

Zbl 1166.53321
Sternheimer, Daniel (ed.) et al., Deformation theory and symplectic geometry. Proceedings of the Ascona meeting, Switzerland, June 17–21, 1996. Dordrecht: Kluwer Academic Publishers (ISBN 0-7923-4525-8/hbk). Math. Phys. Stud. 20, 315-319 (1997).

From the text: The concept of deformation quantization has been defined and exemplified in *F. Bayen, M. Flato, C. Frønsdal, A. Lichnerowicz* and *D. Sternheimer* [Ann. Phys. 111, 61–110, 111–151 (1978; Zbl 0377.53024, Zbl 0377.53025)]. The existence of star-products on every symplectic manifold has been established in *M. De Wilde* and *P. B. A. Lecomte* [Lett. Math. Phys. 7, 487–496 (1983; Zbl 0526.58023)] and in *B. V. Fedosov* [J. Differ. Geom 40, No. 2, 213–238 (1994; Zbl 0812.53034)]. This gives a reasonable physical picture of the noncommutative algebra quantum observables with built-in classical limit. However, the discussion of a formal analogue of representations of the deformed algebra in some 'Hilbert space' seems to have been restricted to examples in the literature up to now. In [Commun. Math. Phys. 195, No. 3, 549–583 (1998; Zbl 0989.53057)] the present authors have proposed how to construct formal pre-Hilbert spaces for the deformed algebra in the same category by means of a generalized Gel'fand-Naimark-Segal (GNS) construction. We now briefly review this construction and apply this in the next section to the Weyl star-product on the cotangent bundle of $\mathbb{R}^n$ (which will give back the usual Schrödinger representation together with the Weyl symmetrization rule; details thereof can be found in the authors' paper (loc. cit) .

The third section contains new material: we describe how to incorporate the usual WKB expansion into the framework of star-products and GNS representations by means of a certain positive linear functional on the deformed algebra having support on a projectable Lagrangian submanifold graph (dS) of $T^*\mathbb{R}^n$. The main trick is to use a suitable form of the star-exponential $e^{*\frac{it}{\lambda}S}$.

For the entire collection see [Zbl 0923.00023].

MSC:

53D55	Deformation quantization, star products	Cited in **6** Documents
81Q20	Semiclassical techniques, including WKB and Maslov methods applied to problems in quantum theory	
81S10	Geometry and quantization, symplectic methods	

Full Text: arXiv