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Overholonomic arithmetic $\mathcal{D}$-modules. ($\mathcal{D}$-modules arithmétiques surholonomes.) (French)

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Let k be a perfect field k of characteristic $p > 0$. The formalism of ℓ-adic sheaves, $\ell \neq p$, has been the main tool for Deligne's proof of the Weil conjectures on Zeta functions of k-varieties for finite fields k. The basic underlying cohomological principle is the the stability of the category of *constructible* ℓ-adic sheaves under Grothendieck's six operations: $\otimes$, $\mathcal{H}om$, f_*, f^*, $f_!$, $f^!$.

In order to also have an effective p-adic cohomological theory for k-varieties (i.e. one taking values in $\mathbb{Q}_p$-vector spaces), Berthelot proposed to use arithmetic $\mathcal{D}$-modules: module sheaves over certain sheaves of differential operators $\mathcal{D}^{\dagger}_{\mathbb{Q}}$ on formal liftings to characteristic zero of the k-varieties in question. He worked out the basic definitions and first properties, and he also suggested far reaching conjectures on the expected overall picture. In particular he defined the concept of *holonomic* $F-\mathcal{D}$-module complexes (where the F indicates the additional action of a Frobenius operator) and conjectured that they play a similar role to those of the *constructible* ℓ-adic sheaves; namely, he conjectured their stability under the relevant six Grothendieck type operations.

Based on his notion of *overcoherent* $\mathcal{D}$-modules developed in earlier papers, the author introduces here his concept of *overholonomic* $\mathcal{D}$-module complexes. He establishes their stability under five of the six Grothendieck type operations: direct images, inverse images, extraordinary inverse images, extraordinary direct images and the duality functor. He also announces their stability under internal and external tensor products for a future paper.

Overholonomic $F-\mathcal{D}$-module complexes are holonomic, so these results prove an essential part of Berthelot's conjecture. Conjecturally, the reverse is true as well, i.e. the categories of holonomic $F-\mathcal{D}$-module complexes and overholonomic $F-\mathcal{D}$-module complexes should be equivalent.

Moreover, the author shows that for a smooth k-variety, overconvergent unit-root F-isocrystals are overholonomic, and in particular holonomic, as also conjectured by Berthelot.

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