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Some recent developments in Lagrangian mean curvature flows. (English) Zbl 1169.53052


In a nicely written survey, the author goes over recent results of the mean curvature flow for Lagrangians with emphasis on global existence and convergence in the smooth category. The mean curvature flow is significantly more difficult to study on submanifolds of co-dimension greater than 1 than when restricted to hypersurfaces. Note, for example, the fact that the normal bundle of a higher co-dimensional submanifold could be highly non-trivial. The results which can be carried over in any dimension and co-dimension are Brakke’s regularity theorem, Hamilton’s maximum principle for tensors, Huisken’s monotonicity formula and White’s regularity theorem. However, other properties cannot be generalized, some as simple as the fact that an embedded hypersurface remains embedded along the mean curvature flow.

Among mean curvature flows on submanifolds of higher co-dimension, two properties suggest a superior behavior in the case of Lagrangian submanifolds. When the ambient space is Kähler-Ricci, being Lagrangian is preserved during the evolution by the mean curvature flow. Perhaps even more important, the normal bundle of a Langrangian submanifold is isometric to the tangent bundle by the almost complex structure $J$ induced by the symplectic structure of the ambient manifold, namely $\omega(X, Y) = \langle JX, Y \rangle$. One section of the paper is devoted to the case when the ambient space is Calabi-Yau and the Thomas-Yau conjecture which is also revisited at the end of the paper. The latter states that, in a Calabi-Yau manifold, a compact embedded Lagrangian submanifold $\Sigma$ evolves under the mean curvature flow for infinite time and converges smoothly as $t \to \infty$ to a special Lagrangian submanifold in the Hamilton isotopy class of $\Sigma$.

For given initial conditions, to understand global existence and convergence of mean curvature flow for Lagrangians, but not only, it is important to study the singularities of the flow. In the curvature flows literature, these are often classified in type I and type II. For example, for the curve shortening flow, which is also the simplest case of Langrangian mean curvature flow, any type I singularity is a shrinking circle, while any type II is the grim reaper solution. They are both evolving homothetically under the flow, which is why they are called self-similar solutions and, in fact, there is a correspondence between singularities and self-similar solutions which was often exploited. In this paper, one will find a subsection of the existence and convergence part of the paper devoted to the construction of self-similar solutions in Lagrangians mean curvature flows.

For the entire collection see [Zbl 1149.53004].

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53C44 \hfill Geometric evolution equations (mean curvature flow, Ricci flow, etc.) (MSC2010) Cited in 8 Documents

53D12 \hfill Lagrangian submanifolds; Maslov index

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