

Zudilin, V. V.

Algebraic relations for multiple zeta values. (English. Russian original) [Zbl 1171.11323](#)
Russ. Math. Surv. 58, No. 1, 1-29 (2003); translation from Usp. Mat. Nauk 58, No. 1, 3-32 (2003).

The following conjecture can be regarded as mathematical folklore with respect to the Riemann zeta function. Conjecture: The numbers $\pi, \zeta(3), \zeta(5), \zeta(7), \dots$ are algebraically independent over \mathbb{Q} . In this survey the author discusses a generalization of the problem of algebraic independence for values of the Riemann zeta-function at positive integers (the so-called zeta values). By means of the present publication the author hopes to attract the attention of (surely not only) Russian mathematicians to problems connected with multiple zeta values. The contents of the paper is best described by its table of contents:

§1. Introduction, §2 Multiple zeta values, §3. Identities: the method of partial fractions, §4. Algebra of multiple zeta values, §5. Shuffle algebra of generalized polylogarithms, §6. Duality theorem, §7. Identities: the generating function method, §8. Quasi-shuffle products, §9. Functional model of the stuffle algebra, §10. Hoffman's homomorphism for the stuffle algebra, §11. Derivations, §12. Ihara-Kaneko derivations and Ohno's relations, §13. Open questions. §14. q -analogues of multiple zeta-values.

This survey contains a wealth of information, clearly presented and with full proofs. A beautiful introductory survey to this very active research topic.

Reviewer: [Olaf Ninnemann \(Berlin\)](#)

MSC:

- [11M32](#) Multiple Dirichlet series and zeta functions and multizeta values
- [11G55](#) Polylogarithms and relations with K -theory
- [11J72](#) Irrationality; linear independence over a field
- [33D70](#) Other basic hypergeometric functions and integrals in several variables

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