Zaldívar, Felipe
Introduction to group theory. (Introducción a la teoría de grupos.) (Spanish) [Zbl 1171.20001]

The book gives an elementary introduction to group theory. Different aspects of the theory are considered with particular emphasis on finite groups. The aim is to prepare students for a higher level on group theory.

The book begins with an attempt to describe the concept of symmetry to motivate the idea of group and then to discuss some important examples: cyclic groups, permutations and matrices. Basic theorems from the Lagrange theorem to the Sylow theorems are introduced and they are applied to an elementary introduction to the study of simple and solvable groups.

The final part of the book uses linear algebra combined with group theory to introduce the reader to the theory of representations of finite groups. These results are applied for proving Burnside’s theorem whose statement says that all finite groups whose orders are of the form $p^aq^b$, with $p$ and $q$ primes, are solvable groups.

At the end of each chapter, the author includes some notes with references that allow the reader to know the historical development of group theory. Moreover, problems to solve are included at the end of each chapter.

The first chapter is devoted to describe the concept of symmetry and to introduce binary operations. In the second chapter, definition of group is given. Examples and basic concepts and definitions are considered.

In chapter three cyclic groups are studied while in chapter four permutations are considered.

Congruences are introduced in chapter five. Normal subgroups, quotient groups and commutators are studied. In chapter six homomorphisms are considered and isomorphism theorems are proved. In chapter seven, direct products and finite Abelian groups are studied.

Chapter eight is devoted to actions of groups and Burnside’s lemma is proved, while in chapter nine, Cauchy and Sylow theorems are considered.

Chapter ten deals with simple groups, while the eleventh one is devoted to solvable groups. Matrices are considered in chapter twelve.

In chapter thirteen, linear representations of finite groups are studied. In chapter fourteen characters of finite groups are considered while applications, like Burnside’s theorem, are studied in chapter fifteen.

The author includes an appendix dedicated to algebraic integers which are used in the proof of Burnside’s theorem.

Reviewer: M. Concepcion López-Díaz (Oviedo)

MSC:
20-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to group theory

Keywords:
group theory; elementary textbook; finite groups