**Griso, Georges**

**Asymptotic behaviour of curved rods by the unfolding method.** (English) Zbl 1174.74313

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Summary: We consider in this work general curved rods with a circular cross-section of radius $\delta$. Our aim is to study the asymptotic behaviour of such rods as $\delta \to 0$, in the framework of the linear elasticity according to the unfolding method. It consists in giving some decompositions of the displacements of such rods, and then in passing to the limit in a fixed domain. A first decomposition concerns the elementary displacements of a curved rod which characterize its translations and rotations, and the residual displacements related to the deformation of the cross-section. The second decomposition concerns the displacements of the middle-line of the rod. We prove that such a displacement can be written as the sum of an inextensional displacement and of an extensional one. An extensional displacement will modify the length of the middle-line, while an inextensional displacement will not change this length in a first approximation. We show that the $H^1$-norm of an inextensional displacement is of order 1, while that of an extensional displacement is in general, of order $\delta$. A priori estimates are established and convergence results as $\delta \to 0$, are given for the displacements. We give their unfolded limits, as well as the unfolded limits of the strain and stress tensors. To prove the convergence of the strain tensor, the introduction of elementary and residual displacements appears as essential. By passing to the limit as $\delta \to 0$ in the linearized system of the elasticity, we obtain on the one hand, a variational problem that is satisfied by the limit extensional displacement, and on the other hand, a variational problem coupling the limit of inextensional displacements and the limit of the angle of torsion.

**MSC:**

74K10 Rods (beams, columns, shafts, arches, rings, etc.)
35B40 Asymptotic behavior of solutions to PDEs
35Q72 Other PDE from mechanics (MSC2000)
74G10 Analytic approximation of solutions (perturbation methods, asymptotic methods, series, etc.) of equilibrium problems in solid mechanics



**Keywords:**

Unfolding method; linear elasticity; beams; curved rods

**Full Text:** DOI

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