Arioli, Gianni; Szulkin, Andrzej; Zou, Wenming

Multibump solutions and critical groups. (English) Zbl 1175.37066

Trans. Am. Math. Soc. 361, No. 6, 3159-3187 (2009).

The topic of the paper is the existence of multibump solutions for Newtonian systems of ordinary differential equations and for semilinear partial differential equations of Schrödinger type. At first a Newtonian system $-\ddot{q}+B(t)q=W_q(q,t)$ is considered where B,W are periodic in t and B is positive definite. Then for an isolated homoclinic solution q_0 with non-trivial critical group multibump solutions are constructed by gluing translates of q_0. An analogous result is shown for a Schrödinger equation $-\Delta u+v(x)u=g(x,u)$ in $\mathbb{R}^N$. Here V,g are periodic in $x_1,\dots,x_N$ and the spectrum satisfies: $\sigma\left(-\Delta+v\right)\subset(0,\infty)$.

Reviewer: Hans-Bert Rademacher (Leipzig)

MSC:

37J45 Periodic, homoclinic and heteroclinic orbits; variational methods, degree-theoretic methods (MSC2010)
34C28 Complex behavior and chaotic systems of ordinary differential equations
35J20 Variational methods for second-order elliptic equations
35Q55 NLS equations (nonlinear Schrödinger equations)

Keywords:

multibump solution; critical group; Bernoulli shift; Newtonian system; Schrödinger equation; critical exponent

Full Text: DOI

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