Michor, Peter W.

Topics in differential geometry. (English) Zbl 1175.53002

Graduate Studies in Mathematics 93. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-2003-2/hbk). xi, 494 p. (2008).

The book under review is a very interesting introduction to the fundamentals of differential geometry by presenting a wide range of topics (manifolds, Lie groups and group actions, differential forms and de Rham cohomology, bundles and connections, Riemannian manifolds, isometric group actions and symplectic and Poisson geometry) in a modern, coordinate-free manner.

A good knowledge of analysis and linear algebra and familiarity with basic topics from classical differential geometry of curves and surfaces, and topology, are required in order to use this book.

Chapter 1 surveys the basic theory of differentiable manifolds. It also includes a series of non-standard results, such as the ones concerning initial submanifolds and distributions of non-constant rank.

In Chapter 2, Lie groups and Lie group actions are presented in detail. Moreover, the basics of invariant theory, as Hilbert-Nagata and Schwarz's theorems, are presented.

Chapter 3 begins with a detailed treatment of vector bundles, and, in particular, of tangent bundles. It proceeds with the theory of differential forms and de Rham cohomology, used in order to compute the cohomology of compact Lie groups. The chapter concludes with a section dedicated to extensions of Lie algebras and Lie groups.

Chapter 4 discusses bundles and connections. It starts with the Frölicher-Nijenhuis bracket and then connections in fiber bundles are presented. A section is dedicated to principal fiber bundles and G-bundles, including the tangent bundles of homogeneous spaces and of Grassmann manifolds, and the bundle of gauges. The characteristic classes (Pontryagin classes, Chern classes, Todd class) and the Atiyah-Singer index formula are also considered.

Chapter 5 contains standard subjects in Riemannian geometry: the geometry of geodesics (first variational formula, the geodesic exponential mapping, Hopf-Rinow Theorem), parallel transport and curvature, Riemannian immersions and submersions and Jacobi fields (Cartan theorem, conjugate points, Myers theorem).

The last chapter deals with symplectic and Poisson geometry, by emphasizing group actions, momentum mappings and reductions.

The concepts are illustrated by various examples and interesting exercises are proposed to the reader.

Reviewer: Cezar Dumitru Oniciuc (Iaşi)

MSC:

53-02 Research exposition (monographs, survey articles) pertaining to differential geometry

53C20 Global Riemannian geometry, including pinching

53D05 Symplectic manifolds (general theory)

53D17 Poisson manifolds; Poisson groupoids and algebroids