Quarteroni, Alfio; **Rozza, Gianluigi**

Numerical solution of parametrized Navier-Stokes equations by reduced basis methods.
(English) Zbl 1178.76238
Numer. Methods Partial Differ. Equations 23, No. 4, 923-948 (2007).

Summary: We apply the reduced basis method to solve Navier-Stokes equations in parametrized domains. Special attention is devoted to the treatment of the parametrized nonlinear transport term in the reduced basis framework, including the case of nonaffine parametric dependence that is treated by an empirical interpolation method. This method features (i) a rapid global convergence owing to the property of the Galerkin projection onto a space W_N spanned by solutions of the governing partial differential equation at N (optimally) selected points in the parameter space, and (ii) the offline/online computational procedures that decouple the generation and projection stages of the approximation process.This method is well suited for the repeated and rapid evaluations required in the context of parameter estimation, design, optimization, and real-time control. Our analysis focuses on: (i) the pressure treatment of incompressible Navier-Stokes problem; (ii) the fulfillment of an equivalent inf-sup condition to guarantee the stability of the reduced basis solutions. The applications that we consider involve parametrized geometries, like e.g. a channel with curved upper wall or an arterial bypass configuration.

MSC:

76M10	Finite element methods applied to problems in fluid mechanics	Cited in **1** Review
65N35	Spectral, collocation and related methods for boundary value problems involving PDEs	Cited in **38** Documents
76D05	Navier-Stokes equations for incompressible viscous fluids	

Keywords:

parametrized partial differential equations; Navier-Stokes equations; reduced basis methods; Galerkin finite element approximation; inf-sup condition; supremizers; empirical interpolation

Full Text: DOI

References:

[1] Barrault, C. R. Acad. Sci. 339 pp 667– (2004) · Zbl 1061.65118 · doi:10.1016/j.crma.2004.08.006

[2] and , Mixed and hybrid finite element methods, Springer-Verlag, New York, 1991. · Zbl 0788.73002 · doi:10.1007/978-1-4612-3172-1

[3] Flow optimization using sensitivities, Notes of School of Computational Science, Florida State University, 2005.

[4] Cahouet, Int J Numer Methods Fluids 8 pp 869– (1988) · Zbl 0654.76020

[5] , , and , Spectral methods: evolution to complex geometries and applications to fluid dynamics, Springer, Heildeberg, 2007.

[6] An introduction to the mathematical theory of the Navier-Stokes equations, Volume II: Nonlinear steady problem, Springer-Verlag, New York, 1994.

[7] and , Finite element methods for Navier–Stokes equations, Springer-Verlag, Berlin, 1986. · Zbl 0585.65077 · doi:10.1007/978-3-642-61623-5

[8] Grepl, SIAM

[9] and , Incompressible flow and the finite elements method, John Wiley, New York, 2000.

[10] Finite element method for viscous incompressible flows: a guide to theory, practice, and algorithms, Academic Press, Boston, 1989.

[11] Perspectives in flow control and optimization, SIAM, Advances in Design and Control, Philadelphia, 2003.

[12] Ito, J Computat Phys 143 pp 403– (1998)

[13] Ito, Math Comput Modell 33 pp 173– (2001)

[14] , and , A reduced basis method for the steady Navier–Stokes problem, in "Reduced basis modelling of hierarchical flow system," PhD Thesis, Norwegian University of Science and Technology, 2006.

[15] , and , Certified real-time solution of parametrized partial differential equations, , and , editors, Handbook of materials modeling, Kluwer Academic, 2005.

[16] Papaharilaou, J Biomech 35 pp 1225– (2002)

[17] Peterson, SIAM J Sci Stat Comput 10 pp 777– (1989)

[18] Prud'homme, J Fluids Eng 172 pp 70– (2002)

[19] Prud'homme, Math Modell Numer Anal 36 pp 747– (2002)

[20] Prud'homme, Comput Visualization Sci 6 pp 147– (2004)

[21] and , Numerical approximation of partial differential equations, Springer-Verlag, Berlin, 1994.

[22] , and , Numerical mathematics, Springer, New York, 2000.

[23] Reduced-basis output bound methods for parametrized partial differencial equations, PhD Thesis, MIT, Massachusetts Institute of Technology, February 2003.

[24] Rozza, Comput Methods Appl Mech Eng 196 pp 1244– (2007)

[25] Rozza, Appl Numer Math 55 pp 403– (2005)

[26] Rozza, Int J Numer Methods Fluids 47 pp 1411– (2005)

[27] Real time reduced basis techniques for arterial bypass geometries, editor, Computational fluid and solid mechanics, Elsevier, 2005, pp. 1283–1287.

[28] Shape design by optimal flow control and reduced basis techniques: applications to bypass configurations in haemodynamics, Ph.D. Thesis, EPFL, Ecole Polytechnique Fédérale de Lausanne, 2005.

[29] Rozza, Comput Visualization Sci

[30] Sherwin, Adv Fluid Mech 34 pp 327– (2003)

[31] Sobey, J Fluid Mech 96 pp 1– (1980)

[32] Stephanoff, J Fluid Mech 96 pp 27– (1980)

[33] Navier–Stokes equations: theory and numerical analysis, North-Holland, Amsterdam, 1984. · Zbl 07.0137.01

[34] Veroy, Int J Numer Methods Fluids 47 pp 773– (2005)

[35] and , Reduced-basis approximation of the viscosity-parametrized incompressible Navier–Stokes equation: rigorous a posteriori error bounds, Proceedings Singapore-MIT Alliance Symposium, January 2004.