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Nonlinear times series: semiparametric and nonparametric methods. (English)

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This book aims at studying some nonlinear models of time series. The work is divided into two parts. The first one deals with discrete-time processes, and the second one considers continuous-time processes.

The book begins, in Chapter one, with an introduction. There the author presents different data sets and models, in which a nonlinear model fits well. Examples of such data sets and models are: global temperature series for 1867–1993, the records of Standard & Poor's 500 for 1958–2005, partially autoregressive models, a population biology model, semiparametric diffusion models and continuous-time models with long-range dependence. In Chapter two, nonlinear models are introduced and several procedures of estimation are considered. In the first place the following partially linear model is studied: Let Y_t and X_t be one-dimensional and d-dimensional time series data, respectively. The conditional mean function $\mathbb{E}[Y_t\,|\,X_t = x]$ may be estimated by the well known Naradaya-Watson method whenever $d \leq 3$. When d is greater than three this estimation method can also be used and its asymptotic theory can be constructed. In practice this procedure may not be recommended, unless the number of data is extremely large, due to a phenomenon known as the curse of dimensionality. This leads to introducing semiparametric additive models. Consider the decomposition $X_t = (U_t^\tau, V_t^\tau)^\tau$, $m(X_t) = \mathbb{E}[Y_t\,|\,X_t]$ and $e_t = Y_t - m(X_t)$. The author defines the semiparametric model as

$$Y_t = \mu + U_t^\tau \beta + g(V_t) + e_t. \tag{1}$$

Here β is a finite-dimensional parameter, linked to the linear part, and g is a nonlinear function that assumes different forms, in view of solving the problem of the curse of dimensionality. In this chapter several methods of estimation are studied, namely: semiparametric series estimation, semiparametric kernel estimation and semiparametric single-index estimation. Each method corresponds to a particular parametric form and a class of functions g. A number of results establish consistency and asymptotic normality of these estimators, both for the linear part and for the nonlinear one. The proofs are deferred to a section named "Technical notes", where the assumptions that must be hold by the components of the processes are given. One of such assumptions is the necessary α-mixing condition to be satisfied by the processes.

Chapter three begins with constructing a test for parametric models. The model is now $Y_t = m(X_t) + e_t$, $t = 1, \dots T$, where $\{X_t\}$ is a sequence of strictly stationary time series and $\{e_t\}$ is a sequence of mean zero and finite second order moment i.i.d errors. The author constructs a procedure to test

$$\begin{aligned} &\mathcal{H}_{01} : m(x) = m_{\theta_0}(x) \text{ versus} \\ &\mathcal{H}_{01} : m(x) = m_{\theta_1}(x) + C_T \Delta(x) \text{ for all} x \in \mathbb{R}^d, \end{aligned}$$

where θ_0 and θ_1 are d-dimensional parameters, C_T is a sequence of real numbers and $\Delta(x)$ is a continuous function. The method consists in defining both a nonparametric estimator $\hat{m}_h(\cdot)$ and a parametric estimator $m_{\hat{\theta}}(\cdot)$ of $m(\cdot)$, and using a test based on the L^2 deviation between these two estimators. Then the author gives an asymptotic level of the test. Some other tests are introduced for semiparametric variance models and other similar models.

In Chapter four the author studies the techniques of model selection for nonlinear time series. In this chapter the semiparametric form (1) is assumed and the problem is to look for optimal subsets of the variables U_t and V_t that fit well the data. Two methods of model selection are considered, first the semiparametric cross-validation and second the semiparametric penalty function method. These two methods are well known for independent data. The case of non-independent data is definitely more difficult and moreover its solution needs some technicalities.

The last two chapters deal with continuous-time processes. Chapter five is dedicated to estimating both the variance and drift parameters in Itô diffusion processes. The processes are observed in a uniform mesh and the asymptotics are taken both when the number T of observations tends to infinity and the size of the observation mesh, $\Delta(T)$, tends to zero when $T \to \infty$. Three types of estimators are considered and their asymptotic properties established. I describe only one of these methods. In this case the procedure consists in estimating firstly the marginal density of the processes by making use of some ergodicity properties, secondly by building up an estimator of the instantaneous variance, and finally by mixing both estimators by using the Fokker-Planck equation for obtaining an estimator of the drift. The last chapter introduces long range dependence (LRD) in Gaussian processes and studies a LRD model in continuous time. Here the estimation is made by a minimum contrast method, similar to that one of Whittle for discrete time processes.

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MSC:

62M10	Time series, auto-correlation, regression, etc. in statistics (GARCH)
62M05	Markov processes: estimation; hidden Markov models
62G08	Nonparametric regression and quantile regression
62-02	Research exposition (monographs, survey articles) pertaining to statistics
60J60	Diffusion processes

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estimation of diffusion processes; estimation of continuous time long range dependence processes

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