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Resolvent estimates for the linearized compressible Navier-Stokes equation in an infinite layer. (English) Zbl 1180.35413

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The author deals with the following resolvent problem: (1) $(\lambda + L)u = f$ in an infinite layer $\Omega = \mathbb{R}^{n-1}x(0,a)$, $n \geq 2$, where $\lambda \in \mathbb{C}$ is a parameter, $f = f(x)$ is a given function with values in $\mathbb{R}^{n+1}$, $u =^T (\varphi, m)$ is the unknown function with $\varphi = \varphi(x) \in \mathbb{R}$ and $m =^T (m^1(x), \dots, m^n(x)) \in \mathbb{R}^n$, and L is an operator defined by

$$L = \begin{pmatrix} 0 & \gamma \mathrm{div} \\ \gamma\nabla & -\nu\Delta I_n - \widetilde{\nu}\nabla\mathrm{div} \end{pmatrix}$$

with positive constants ν and γ and a nonnegative constant $\widetilde{\nu}$. Here $x =^T (x', x_n) \in \Omega$ with $x' \in \mathbb{R}^{n-1}$, $x_n \in (0,a)$; the superscript $T(x', x_n)$ stands for the transposition; I_n is the $(n \times n)$ identity matrix; and div, ∇ and Δ are the usual divergence, gradient and Laplacian with respect to x. The author considers (1) under the boundary condition $m|_{\partial\Omega} = 0(2)$. The author establishes the L^p estimates for the solution of (1)–(2) for $1 \leq p \leq \infty$. The estimates show that $-L$ generates an analytic semigroup in $W^{1,p} \times L^p$ for $1 < p < \infty$. Based on the estimates the author obtains short-time estimates for the semigroup in L^p norms for all $1 \leq p \leq \infty$. Moreover, the author establishes the estimates for the high frequency part of the resolvent, which lead to the exponential decay of the corresponding part of the semigroup.

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MSC:

35Q30 Navier-Stokes equations
76N15 Gas dynamics (general theory)

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