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**Existence and uniqueness of positive and nondecreasing solutions for a class of singular fractional boundary value problems.** (English) Zbl 1182.34005

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From the introduction: We discuss the boundary-value problem

$$\begin{aligned} D_{0+}^{\alpha} u(t) + f(t, u(t)) &= 0, & 0 < t < 1, \\ u(0) = u'(1) = u''(0) &= 0, \end{aligned} \quad (*)$$

where  $2 < \alpha \leq 3$ ,  $D_{0+}^{\alpha}$  is the Caputo's differentiation and  $f : (0, 1] \times [0, \infty) \rightarrow [0, \infty)$  with  $\lim_{t \rightarrow 0+} f(t, -) = \infty$  (i.e.,  $f$  is singular at  $t = 0$ ).

We prove the existence and uniqueness of a positive and nondecreasing solution for the problem (\*) by using a fixed point theorem in partially ordered sets.

**MSC:**

**34A08** Fractional ordinary differential equations

**34B15** Nonlinear boundary value problems for ordinary differential equations

**47N20** Applications of operator theory to differential and integral equations

Cited in **24** Documents

**Full Text:** [DOI](#) [EuDML](#)

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