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An estimate of the H^1-norm of deformations in terms of the L^1-norm of their Cauchy-Green tensors. (English) Zbl 1183.74008

C. R., Math., Acad. Sci. Paris 338, No. 6, 505-510 (2004).

Summary: Let Ω be a bounded open connected subset of $\mathbb{R}^n$ with a Lipschitz-continuous boundary and let $\mathbf{\Theta} \in \mathcal{C}^1(\overline{\Omega}; \mathbb{R}^n)$ be a deformation of the set $\overline{\Omega}$ satisfying $\det \nabla\mathbf{\Theta} > 0$ in $\overline{\Omega}$. It is established that there exists a constant $C(\mathbf{\Theta})$ with the following property: for each deformation $\mathbf{\Phi} \in H^1(\Omega; \mathbb{R}^n)$ satisfying $\det \nabla\mathbf{\Phi} > 0$ a.e. in Ω, there exist an $n \times n$ rotation matrix $\mathbf{R} = \mathbf{R}(\mathbf{\Phi}, \mathbf{\Theta})$ and a vector $\mathbf{b} = \mathbf{b}(\mathbf{\Phi}, \mathbf{\Theta})$ in $\mathbb{R}^n$ such that

$$\left\| \mathbf{\Phi} - (\mathbf{b} + \mathbf{R}\mathbf{\Theta}) \right\|_{\mathbf{H}^1(\Omega)} \leq C(\mathbf{\Theta}) \left\| \nabla\mathbf{\Phi}^T \nabla\mathbf{\Phi} - \nabla\mathbf{\Phi}^T \nabla\mathbf{\Theta} \right\|_{\mathbf{L}^1(\Omega)}^{1/2}.$$

The proof relies in particular on a fundamental 'geometric rigidity lemma', recently proved by G. Friesecke, R. D. James, and S. Müller.

MSC:

74A05 Kinematics of deformation

Full Text: DOI

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