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An introduction to random matrices. (English) Zbl 1184.15023

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Properties of random matrices and especially properties of their eigenvalues are important in many fields of science and also in pure mathematics.

The goal of the given book is to present a rigorous introduction to the basic theory of random matrices, including free probability, to be accessible to graduate students in mathematics or related sciences who have mastered probability theory at the graduate level, but not necessarily the advanced fields of functional analysis, algebra and geometry. The book is divided into five chapters and several appendices.

In the first three chapters after a short introduction the real and complex Wigner matrices are considered. Wigner's Theorem asserts that under appropriate assumption on the law of entries the empirical measure of eigenvalues of Wigner matrices converges towards a deterministic probability measure, the semicircle law. Several proofs of Wigner's Theorem are given, among them one using combinatorial techniques which are also used to yield central limit theorems and estimates of the spectral radius of Wigner matrices. The Stieltjes transform of measures is defined and used to provide another proof of Wigner's Theorem. After an analysis of matrix properties with entries distributed according to general laws special situations involving additional symmetry are discussed. The simplest of these are the Gaussian orthogonal ensemble (GOE) and the Gaussian unitary ensemble (GUE). Their extra symmetry is important in deriving an explicit joint distribution for the eigenvalues. The expression for the joint density of the eigenvalues for GUE in terms of a determinant involving appropriate orthogonal polynomials is needed for the study of the gap probability at 0, that is the probability that no eigenvalue is present in an interval around the origin.

In Chapters 4 and 5 more advanced techniques are used requiring more extensive background to introduce the theory of free probability. At first, a re-derivation of the joint law of the eigenvalues of the Gaussian ensemble in a geometric framework based on Lie theory is presented. Within this framework the expressions for the joint distributions of eigenvalues of Wishart matrices and of random matrices from the various unitary groups are derived. Then, free probability theory, a theory for certain noncommutative variables including a notion called free independence, is discussed. Some definitions are introduced and links with random matrices are analysed.

Some background material such as basic results and definitions from linear algebra, topology, probability measures on Polish spaces, large deviation theory, the skew field of quaternions, manifolds, operator algebras and stochastic analysis is presented in the appendices. Each chapter is closed with bibliographical notes covering the given subject in chronological order.

Reviewer: Václav Burjan (Praha)

MSC:

15-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to linear algebra
15B52	Random matrices (algebraic aspects)
15A18	Eigenvalues, singular values, and eigenvectors
60B20	Random matrices (probabilistic aspects)

Keywords:

Wigner matrices; joint distributions of eigenvalues; Gaussian ensembles; Fredholm determinants; Tracy-Widom distribution; gap probability; free probability; monograph; random matrices; semicircle law; central limit theorems; spectral radius; Stieltjes transform; Wigner's Theorem; Wishart matrices