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Local geometric Langlands correspondence and affine Kac-Moody algebras. (English)

Zbl 1184.17011
Ginzburg, Victor (ed.), Algebraic geometry and number theory. In Honor of Vladimir Drinfeld's 50th birthday. Basel: Birkhäuser (ISBN 978-0-8176-4471-0/hbk). Progress in Mathematics 253, 69-260 (2006).

Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{C}$ and G a connected algebraic group with Lie algebra $\mathfrak{g}$. The affine Kac-Moody algebra $\hat{\mathfrak{g}}$ is the universal central extension of the formal loop algebra $\mathfrak{g}((t))$. Representations of $\hat{\mathfrak{g}}$ have a parameter, an invariant bilinear form on $\mathfrak{g}$, which is called the level. Representations corresponding to the bilinear form which is equal to minus one half of the Killing form are called representations of critical level. Such representations can be realized in spaces of global sections of twisted D-modules on the quotient of the loop group $G((t))$ by its "open compact" subgroup K, such as $G[[t]]$ or the Iwahori subgroup I.

This long paper is the first in a series devoted to the study of the categories of representations of the affine Kac-Moody algebra $\hat{\mathfrak{g}}$ of the critical level and D-modules on $G((t))/K$ from the point of view of a geometric version of the local Langlands correspondence. What this means is nicely explained in the Introduction.

The paper is divided into five parts. Part I reviews results concerning opers and Miura opers. In Part II the authors discuss various categories of representations of affine Kac-Moody algebras at the critical level. The authors then give more precise formulations of certain conjectural equivalences and the interrelations between them. In particular, they prove one of their main results, Theorem 8.17: The functor $L\Gamma^{\mathrm{Hecke}}$, restricted to $D^b(\mathfrak{D}(\mathrm{Gr}G)^{\mathrm{Hecke}}_{\mathrm{crit}}\text{-mod})$, is fully faithful.

In Part III Wakimoto modules are reviewed. The authors give a definition of Wakimoto modules by means of a kind of semi-infinite induction functor and also describe various important properties of these modules.

In Part IV they prove the Main Theorem 6.9, describing the (bounded) derived category of $\hat{\mathfrak{g}}_{\mathrm{crit}}\text{-mod}_{\mathrm{nilp}}$ as follows:

$$D^b(\hat{\mathfrak{g}}_{\mathrm{crit}}\text{-mod}_{\mathrm{nilp}})^{I,m} \simeq D^b(\mathrm{QCoh}(\tilde{\check{\mathfrak{g}}}/\check{G} \times_{\check{\mathfrak{g}}/\check{G}} \mathrm{Op}^{\mathrm{nilp}}_{\check{\mathfrak{g}}})),$$

where $\tilde{\check{\mathfrak{g}}} \to \check{\mathfrak{g}}$ is Grothendieck's alteration. For notations see the original.

Part V is an appendix, most of which is devoted to the formalism of group action on categories.

For the entire collection see [Zbl 1113.00007].

Reviewer: Olaf Ninnemann (Berlin)

MSC:

17B67	Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras
11G45	Geometric class field theory
14D20	Algebraic moduli problems, moduli of vector bundles
22E57	Geometric Langlands program: representation-theoretic aspects

Keywords:

Local geometric Langlands correspondence; affine Kac-Moody algebras; opers; Miura opers; categories of representations; Wakimoto modules; semi-infinite induction functor; conjectural equivalence of categories

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