

**Talagrand, Michel**

**Maharam's problem.** (English) Zbl 1185.28002  
*Ann. Math. (2)* 168, No. 3, 981-1009 (2008).

This paper settles in the negative several long standing conjectures in measure theory. The author constructs (1) a non-zero exhaustive submeasure  $\nu$  on the algebra  $B$  of clopen subsets of the Cantor set that is not absolutely continuous with respect to a measure on  $B$ . Furthermore, no non-zero measure on  $B$  is absolutely continuous with respect to  $\nu$ . This construction gives a negative answer to a problem of *D. Maharam* [*Ann. Math. (2)* 48, 154–167 (1947; [Zbl 0029.20401](#))].

The author's work yields other important results. He proves (2) there exists a  $\sigma$ -complete algebra that satisfies the countable chain condition and is weakly distributive but is not a measure algebra. This gives a counterexample to a conjecture of von Neumann of 1937 [cf. *R. D. Mauldin* (ed.), "The Scottish book. Mathematics from the Scottish Cafe" (Birkhäuser, Boston–Basel–Stuttgart) (1981; [Zbl 0485.01013](#))]. (3) There exists an exhaustive measure that does not have a control measure, a negative solution to the Control Measure Problem.

The paper contains a good presentation of crucial background material due to J. W. Roberts and I. Farah, making it quite self-contained and readable.

Reviewer: [D. R. Bell](#) (Jacksonville)

**MSC:**

[28A12](#) Contents, measures, outer measures, capacities

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