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Decay for the wave and Schrödinger evolutions on manifolds with conical ends. I. (English)

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For d-dimensional ($d \geq 1$) compact Riemannian manifold $\Omega \subset \mathbb{R}^N$ and $(d+1)$-dimensional Riemannian manifold $\mathcal{M} := \{(x, r(x)\omega) : x \in \mathbb{R}, \omega \in \Omega\}$ with $r > 0$ smooth metric $ds^2 = (1 + r'^2(x))dx^2 + r^2(x)ds^2_\Omega$ and conical ends $r(x) = |x| + O(x^{-1})$ at $x \to \pm\infty$ the Hamiltonian flow on them exhibits trapping. The author has obtained the dispersive estimates for the Schrödinger evolution $e^{it\triangle_\mathcal{M}}$ and the wave evolution $e^{it\sqrt{-\triangle_\mathcal{M}}}$ for data of the form $f(x,\omega) = Y_n(\omega)u(x)$, where Y_n are eigenfunctions of $-\Delta_\Omega$ corresponding to eigenvalues μ_n^2.

In the part I of the article the case $d = 1, Y_0 = 1$ is investigated. Two main results are obtained here:

(A) A detailed scattering analysis of Schrödinger operators of the form $-\partial_\xi^2 + V(\xi)$ on the line where $V(\xi)$ has inverse square behavior at infinity.

(B) Estimation of oscillatory integrals by (non)stationary phase.

Reviewer: Boris V. Loginov (Ul'yanovsk)

MSC:

35J10	Schrödinger operator, Schrödinger equation
58J05	Elliptic equations on manifolds, general theory
58J60	Relations of PDEs with special manifold structures (Riemannian, Finsler, etc.)
35P25	Scattering theory for PDEs

Keywords:

Schrödinger evolutions on manifolds; manifolds with conical ends

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