Morgan, Kerri; Farr, Graham
Certificates of factorisation for chromatic polynomials. (English) Zbl 1186.05056

Summary: The chromatic polynomial gives the number of proper $\lambda$-colourings of a graph $G$. This paper considers factorisation of the chromatic polynomial as a first step in an algebraic study of the roots of this polynomial. The chromatic polynomial of a graph is said to have a chromatic factorisation if $P(G, \lambda) = P(H_1, \lambda)P(H_2, \lambda)/P(K_r, \lambda)$ for some graphs $H_1$ and $H_2$ and clique $K_r$. It is known that the chromatic polynomial of any clique-separable graph, that is, a graph containing a separating $r$-clique, has a chromatic factorisation. We show that there exist other chromatic polynomials that have chromatic factorisations but are not the chromatic polynomial of any clique-separable graph and identify all such chromatic polynomials of degree at most 10. We introduce the notion of a certificate of factorisation, that is, a sequence of algebraic transformations based on identities for the chromatic polynomial that explains the factorisations for a graph. We find an upper bound of $n^22^{n^2/2}$ for the lengths of these certificates, and find much smaller certificates for all chromatic factorisations of graphs of order $\leq 9$.

MSC:
05C15 Coloring of graphs and hypergraphs
05C75 Structural characterization of families of graphs
68R10 Graph theory (including graph drawing) in computer science

Keywords: chromatic polynomials; factorisation; chromatic factorisation

Full Text: EuDML EMIS