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Zeros of the alternating zeta function on the line $\operatorname{Re}(s) = 1$. (English) Zbl 1187.11031
Am. Math. Mon. 110, No. 5, 435-437 (2003).

Summary: The alternating zeta function $\zeta^*(s) = 1 - 2^{-s} + 3^{-s} - \dots$ is related to the Riemann zeta function by the identity $(1 - 2^{1-s})\zeta(s) = \zeta^*(s)$. We deduce the vanishing of $\zeta^*(s)$ at each nonreal zero of the factor $1 - 2^{1-s}$ without using the identity. Instead, we use a formula connecting the partial sums of the series for $\zeta^*(s)$ to Riemann sums for the integral of x^{-s} from $x = 1$ to $x = 2$. We relate the proof to our earlier paper "The Riemann Hypothesis, simple zeros, and the asymptotic convergence degree of improper Riemann sums," *Proc. Am. Math. Soc.* 126, No. 5, 1311-1314 (1998; [Zbl 0890.11025](#)).

MSC:

[11M41](#) Other Dirichlet series and zeta functions
[11M06](#) $\zeta(s)$ and $L(s, \chi)$

Cited in **3** Documents

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