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Converse theorem on equivariant genera. (English. Russian original) Zbl 1187.57025

From the introduction: The Hirzebruch genus, that is, a homomorphism \( h : \Omega^* \to R \), where \( \Omega^* = U^*(pt) \) stands for the complex cobordism ring and \( R = \mathbb{Q}, \mathbb{R}, \text{or } \mathbb{C} \) (the ring of rational, real, or complex numbers), is defined from the point of view of characteristic classes by a series \( H(t) \in R[[t]] \) such that \( h([\mathbb{C}P^n]) = [H(t)^{n+1}]_n \).

For any genus \( h : \Omega^* \to R \) and any group \( G \) one can construct an equivariant genus \( h_G : \Omega_G \to K(BG) \otimes R \) which is a homomorphism from the cobordism ring of \( G \)-manifolds into the ring \( K(BG) \otimes R \). A genus \( h \) is said to be rigid if the image of the homomorphism \( h_G \) belongs to the subring \( R \subset K(BG) \otimes R = R[[t]] \) for \( G = S^1 \).

Let \( x, y \in R \) and let \( x + y \) be a positive integer. Then the \( T_{x,y} \)-genus is defined by \( T_{x,y}([\mathbb{C}P^n]) = (x^{n+1} - (-y)^{n+1})/(x + y) \). I. M. Krichever [Math. USSR, Izv. 8, 1271–1285 (1974); translation from Izv. Akad. Nauk SSSR, Ser. Mat. 38, 1289–1304 (1974; Zbl 0315.57021)] proved that the \( T_{x,y} \)-genus is rigid, which led to a new proof of the Atiyah-Hirzebruch formula for the fixed points of \( S^1 \)-manifolds. The objective of the present note is a theorem converse to Krichever’s theorem.

Theorem. If a genus \( h : \Omega^* \to R \) is rigid and \( g_h^{-1}(\log z) \) is a rational function of \( z \), then \( h \) is a \( T_{x,y} \)-genus.

MSC:
57R20 Characteristic classes and numbers in differential topology
58J26 Elliptic genera
53C99 Global differential geometry
57R85 Equivariant cobordism

Full Text: DOI