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**Some formulas for Apostol-Euler polynomials associated with Hurwitz zeta function at rational arguments.** (English) [Zbl 1189.11012](#)

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In [*Pac. J. Math.* 1, 161–167 (1951; [Zbl 0043.07103](#))] *T. M. Apostol* introduced a generalization of the classical Euler polynomials as analogous definition of Apostol type for the so-called Apostol-Euler numbers and polynomials of higher order. The generalization, the Apostol-Euler polynomials  $\mathcal{E}_n^{(\alpha)}(x; \lambda)$ , is defined by means of the following generating function:

$$\left(\frac{2}{\lambda e^z + 1}\right)^\alpha e^{xz} = \sum_{n=0}^{\infty} \mathcal{E}_n^{(\alpha)}(x; \lambda) \frac{z^n}{n!} \quad (|z| < |\log(-\lambda)|),$$

with:  $E_n^{(\alpha)}(x) = \mathcal{E}_n^{(\alpha)}(x; 1)$  and  $\mathcal{E}_n^{(\alpha)}(\lambda) := 2^n \mathcal{E}_n^{(\alpha)}(\alpha/2; \lambda)$ ;  $\mathcal{E}_n(x; \lambda) = \mathcal{E}_n^{(1)}(x; \lambda)$  and  $\mathcal{E}_n(\lambda) := 2^n \mathcal{E}_n(\alpha/2; \lambda)$ , where  $\mathcal{E}_n(\lambda)$ ,  $\mathcal{E}_n^{(\alpha)}(\lambda)$  and  $\mathcal{E}_n(x; \lambda)$  denote the so-called Apostol-Euler numbers, Apostol-Euler numbers of order  $\alpha$  and Apostol-Euler polynomials, respectively. In a similar manner the Apostol-Bernoulli polynomials of order  $\alpha$ , a generalization of the classical Bernoulli polynomials, is introduced. The author derives some relationships between these polynomials and the generalized Hurwitz–Lerch zeta-function. He also gives an explicit series representations for the polynomials involving the Hurwitz zeta-function and the Riemann zeta-function.

Reviewer: Dragan Stankov (Beograd)

**MSC:**

- [11B68](#) Bernoulli and Euler numbers and polynomials
- [11M35](#) Hurwitz and Lerch zeta functions
- [11B73](#) Bell and Stirling numbers
- [33C45](#) Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

Cited in **16** Documents

**Keywords:**

[Bernoulli polynomials](#); [Apostol-Bernoulli polynomials](#); [Gaussian hypergeometric function](#); [Stirling numbers of the second kind](#); [Hurwitz zeta function](#); [Lerch functional equation](#)

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