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Matrix-valued polynomials generated by the scalar-type Rodrigues’ formulas. (English)

The properties of matrix-valued (M-valued) polynomials generated by Rodrigues’ formula
\[ P_n(x) = W(x)^{-1}d^n/dx^n[Q(x)^nW(x)] \]
are analyzed, where \( W(x) \) is an M-valued function satisfying the Pearson equation
\[ Q(x)W(x)^{-1}W'(x) = xL_1 + L_2, \]
with \( L_1, L_2 \in \mathcal{M} \), and \( Q(x) = \sigma x^2 + \tau x + \delta \) (\( L_1 + k\sigma \) is invertible
in \( \mathcal{M} \) for \( k \in \mathbb{N} \)). A general representation of these polynomials in terms of products of simple differential
operators is found. The recurrence relation
\[ xP_n(x) = P_{n+1}(x)\alpha_n + P_n(x)\beta_n + P_{n-1}(x)\gamma_n \]
(\( \alpha_n, \beta_n, \gamma_n \in \mathcal{M} \) are uniquely determined), the leading coefficients of
\( P_n(x) \), and completeness are established. Moreover,
in the commutative case (if \( L_1 \) and \( L_2 \) commute) the ladder relation
\[ \mathcal{L}_n P_n(x) = g_n P_{n-1}(x) \]
is proved, where the first order linear differential operator \( \mathcal{L}_n \) has the form
\[ \mathcal{L}_n = Ax + B + CQ(x)\partial_x \]
and \( g_n \) is a constant is proved. Commutative classes of orthogonal polynomials, corresponding to the weight \( W(x) \),
which is self-adjoint but not positive semidefinite, are found. They satisfy the general properties usually
associated with orthogonal polynomials, and are not of scalar type.

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MSC:
33C50 Orthogonal polynomials and functions in several variables expressible
in terms of special functions in one variable  
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi,
Laguerre, Hermite, Askey scheme, etc.)  
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric
harmonic analysis

Keywords:
orthogonal polynomials; matrix-valued polynomials; Rodrigues’ formulas

Full Text: DOI arXiv

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