zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Sauer, Tilman (ed.); **Majer, Ulrich (ed.)**; **Schirrmacher, Arne (ed.)**; **Schmidt, Heinz-Jürgen (ed.)**
[Hilbert, David]
David Hilbert's lectures on the foundations of physics, 1915-1927. Relativity, quantum theory and epistemology. In collaboration with Arne Schirrmacher and Heinz-Jürgen Schmidt.
(English) Zbl 1190.01027
David Hilbert's Lectures on the Foundations of Mathematics and Physics, 1891–1933 5. Berlin: Springer (ISBN 978-3-540-20606-4/hbk; 978-3-540-68242-4/ebook). xii, 795 p. (2009).

David Hilbert (1862-1943) was an outstanding mathematician whose Gesammelte Abhandlungen (Collected Works) were published during his lifetime, between 1932 and 1935 in three volumes. In addition to these volumes an abundant Nachlass – including lecture concepts, notices, note books written by Hilbert, and lecture notes (Mitschriften) picked up and polished by students and assistants, as well as letters all excellently preserved and almost complete collected in the Department Handschriftenabteilung (Special Collections) of the Library of the University of Göttingen. Although this collection has been available since the 1960s, we have seen only a few publications from it. For example, some lecture notes published by the Mathematical Institute at Göttingen and Laubenbacher's English translation of a lecture. During the digital era, the Digitalisierungszentrum Göttingen has offered about 800 entries of printed papers by Hilbert.

A Hilbert edition has been in existence for about twenty years with the intention of publishing Hilbert's Foundational Lectures in six volumes. This edition is guided by four general editors and special volume editors, with additional staff and collaborators. Their activities have included many conferences and meetings that have taken place in the USA and Europe. The second volume of the edition has now been released: the voluminous Lectures on the Foundations of Physics, 1915-1927 (vol. 5 of the edition). Its 795 pages cover an introduction (24 pp.), six chapters (682 pp. altogether), a table of Hilbert's lecture courses 1886–1934 (18 pp.), a bibliography (40 pp.), and an index (28 pp.).

This edition faces two problems: i) to explain why the edition is restricted to foundational lectures (and furthermore, what foundational lectures are); ii) which sources were chosen and why.

i) More than any other mathematician Hilbert regarded the sciences as a whole, i.e., also including mathematics as a unit. So one needs an explanation of why and how one excludes selections. A closer inspection shows that the concept of foundational lectures is not as easy as it looks at a first glance.

Hilbert never changed his conviction that in physics histrionic mathematical abilities are necessary. Mathematical physics is difficult, because it is a mixture of mathematics and physics rolled into one; geometry is easier and clearer (presented in 2004, in vol. 1 of this edition) and is to be supposed for this volume. In the lecture on the "Foundation of Physics" (1916), Hilbert declared that like the mathematicians who necessarily became philosophers because of set theory, physicists necessarily have to become mathematicians because of the theory of relativity (not noted in the lecture notes used in chapter 2, but in a different version from the Staatsbibliothek Berlin).

For the theory of relativity as well as for quantum mechanics Hilbert regarded the calculus of variations with the help of which one can formulate extremal principles as absolutely inevitable ("as ABC", in the lecture "Mechanik und Gravitationstheorie", 1920). Felix Klein even complained in a letter to W. Pauli that Hilbert overused variational principles for the presentation of physics. Hilbert himself lectured three times on the calculus of variations (1899, 1904, 1915), and in all his foundational lectures as well as in lectures on differential equations (for example 1907, 1909, 1912, 1915) he gave an extensive survey of it, or at least a short one. In his lecture course on quantum mechanics (1922) Hilbert used half of the lecture for the calculus of variations. Finally the subject was so important to his disciple Max Born that he included an overview of Hilbert's 1904 lecture as an appendix in his Principles of Optics (even in 1959!). A further example: in Hilbert's notebook there is an important unpublished entry concerning a general field theory for a proof of sufficiency for strong extrema (see p. 524, fn. 19; fields on pp. 518f., 559f.) which was independently reinvented by *A. R. Forsyth* [The calculus of variations. Cambridge, University Press (1927; JFM 53.0480.01)], *H. Boerner* [Math. Z. 39, 492–500 (1935; Zbl 0010.30603; JFM 61.0549.02)], *R.*

© 2023 FIZ Karlsruhe GmbH

Klötzler [Mehrdimensionale Variationsrechnung. Mathematische Reihe. 44. Basel-Stuttgart: Birkhäuser Verlag; Berlin: VEB Deutscher Verlag der Wissenschaften. 299 S. (1970; Zbl 0199.42901)] and *A. D. Ioffe* and *V. M. Tikhomirov* [Theory of extremal problems [in Russian] (1974; Zbl 0292.90042)]. So one cannot overrate the calculus of variations in Hilbert's foundation of physics and the reader could be entitled to expect that the calculus of variation (and to a certain extent also the theory of integral equations) be rolled into the foundational lectures, at least this branch would make it easier to understand the foundational way. However, nobody on the entire staff is a mathematician, an obvious defect which is reflected in the underrating of the variational technique. Today without the knowledge of the basic principles and the most important methods of the calculus of variations one cannot manage with theoretical physics, Hilbert declared in a lecture on quantum mechanics in 1926 (see p. 609). The editors should take notice of Hilbert's conviction in the forthcoming volume of the "Foundation of Physics, 1898-1914".

ii) In contrast to other editions from the Special Collection of Göttingen's Library, Hilbert's Nachlass is easily available and one can almost argue that there are too many sources. At any rate, Hilbert's handwriting is clear as is the handwriting of his helpful wife Käthe (concept of lectures, notes, letters). Thanks to an old demand by Felix Klein several versions of lecture notes exist for every lecture Hilbert ever delivered: all handwritten lecture notes are easy to read and many of them are even typewritten. Four facsimiles give evidence of this fact but one would have wished to see pages which Hilbert improved and revised (as in Cod. Ms. Hilbert 562 "Mechanik und neue Gravitationstheorie", p. 27; let alone Cod. Ms. 607, p. 24f.). How to make a good selection might be tricky, however in general we have at least one set of lecture notes which was corrected and improved by Hilbert himself. At the end of each document the volume editor gives a detailed description of the item but it is to be regretted that other related sources are not mentioned let alone summarized.

This volume has three parts, dealing with General Relativity (chapters 1–3), epistemological questions (chapter 4), and Quantum Mechanics (chapters 5–6). The book starts with a general introduction, each chapter has a short comment and a description of the sources. The language of the edition is English but Hilbert's texts are given in the original German language. One may wonder why the rather short comments are given in English because only a reader with a thorough command of German will be able to fully profit from the book and such a reader could surely also read German comments. In science, English is regarded as lingua franca; this might be the reason the editors chose the fashionable English, bearing in mind Chesterfield's saying: "If you are not in fashion you are nobody" (1750).

Chapter 1 contains Hilbert's important papers "Grundlagen der Physik, I & II" as they were published in the Göttinger Nachrichten in 1915 and 1917 respectively [Gött. Nachr. 1915, 395–407 (1915; JFM 45.1111.01); ibid. 1917, 53–76 (1917; JFM 46.1298.01); these versions can be seen online as well as in English translations]. In view of priorities in the theory of relativity, these papers are of special interest. There are several controversial opinions, but controversial literature (for example D. Wünsch) is not mentioned. However, T. Sauer gives a fair report on the dramatic events of November 1915 in which Einstein came into play. A crucial role is played by the proofs for the first paper, which are in the Hilbert files in the Special Collections of the Göttingen University Library. Unfortunately, the different version in the proofs is given not until chapter 3 but for convenience it should have been synopsized in print (i.e., from page to page).

Chapter 2 offers a lecture on the same topic "Grundlagen der Physik, I & II" delivered in 1916-1917, picked up by Bär and Scherrer; there are (about six) further lectures on this topic which are not taken into account.

Chapter 3 includes the previously mentioned proofs of the paper "Grundlagen der Physik, I" (Erste Korrektur meiner ersten Note [i.e., First correction of my first communication], 1915). In addition, there are some epistemological questions: a lecture on Space and Time delivered in Bucharest in 1918, as well as short notices Über das Kausalitätsprinzip in der Physik (On the principle of causality) as well as some notices concerning foundational questions on a single sheet of paper.

Chapter 4 deals again with epistomological problems which Hilbert dealt with in his lecture "Natur und mathematisches Erkennen" (Nature and mathematical knowledge; Copenhagen 1921) as well as the untitled handwritten continuation on foundational questions on physics (Copenhagen and Hamburg 1921, three booklets).

Chapter 5 contains a part of a lecture course on the theory of radiation delivered in 1912 (typewritten and probably supervised by E. Hecke). After having finished his theory of integral equations, Hilbert expanded his interest into physics, the application of which he demonstrates in this course, furthermore he used principles to create a variational theory.

© 2023 FIZ Karlsruhe GmbH

Chapter 6 gives the last two lectures Hilbert delivered on Quantum Theory "Mathematische Grundlagen der Quantentheorie" (Mathematical foundations of quantum theory) and "Mathematische Methoden der Quantentheorie" (Mathematical methods of quantum theory) delivered in 1922 and 1926 respectively. The latter course was held immediately after Heisenberg's new quantum mechanics, and Hilbert was very disappointed that in Göttingen his mathematical colleagues did not attend Heisenberg's pioneering lecture.

In the footnotes the editors have made some historical errors: a general derivation of the Euler-Lagrange equation is due to Euler (1744 and elegantly 1771), and to Lagrange in a formal way already in 1755, i.e., before his Mécanique (1788), p. 510, fn. 4; the principle of the last multiplier was already given by Jacobi in 1842 (Vorlesungen über Dynamik, 1866; Chelsea Reprint 1969), p. 521, fn. 1.

Hilbert presents the calculus of variations in the Lagrangian and the dual Hamiltonian formalism. In mechanics the integrand of a variational integral is usually denoted as Lagrange function L whereas the Legendre transform of L is denoted as the Hamilton function H. In his lectures on quantum mechanics Hilbert introduces some new designations, for example he calls the Legendre transform of L the Legendre function (= Hamiltonian) confusing himself as is indicated at p. 526, fn. 21 and later. In addition Hilbert also improves the arguments of functions. Here the editor should have referred to the usual names and characterize Hilbert's designations as non-standard, all the more as in other parts of this volume Hamiltonians appear (p. 289) and the denoting letters are non-uniform in this lecture. Indulging in changes Hilbert tried to introduce the variational concept "quantrix" (p. 518) analogously to eikonal, geodesic distance etc., but in vain. Finally, in the 1926-lecture he spoke of s-surfaces, but in printing this lecture the editors omitted the calculus of variations (p. 609, fn. 4).

Furthermore, the book contains a list of Hilbert's lecture courses (1886-1934). An index and literature is added. The bibliography contains about 500 titles, of which 53 are Hilbert's. It might have been preferable to put Hilbert's works in a separate section from the secondary sources. The last two decades, during which this edition was prepared, are represented only by about 40 and 45 entries respectively. At the end of the book (pp. 723-726) a supplementary list of manuscripts is given, containing 25 further items in 11 libraries different from those of Göttingen. Of course, such lists are always incomplete but it is surprising that, for example, the editors did not note a further copy of the quantum mechanics lecture from 1922 in the Staatsbibliothek in Berlin the same town in which one collaborator works. But much more surprising is the remark: "In the most cases, the editors are not acquainted with the documents [of the supplementary list] themselves" (p. 723). That means that this is not a historical-critical edition. On the other hand, because of the price students will not be able to afford this book, much less the whole edition. (By the way, this edition does not note the lifespans of individuals mentioned in the text!) So this volume inhabits a place somewhere between a definitive critical history and a student edition, although readers of this beautifully printed and well bound book will no doubt appreciate having some important and interesting papers by Hilbert between its covers.

Reviewer: Rüdiger Thiele (Halle)

MSC:

01A75 Collected or selected works; reprintings or translations of classics

Cited in **1** Review
Cited in **2** Documents

Keywords:

collected works; Hilbert; physics; foundational lectures

Full Text: DOI

© 2023 FIZ Karlsruhe GmbH