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Combinatorics and cluster expansions. (English) Zbl 1191.82009

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Summary: This article is about the connection between enumerative combinatorics and equilibrium statistical mechanics. The combinatorics side concerns species of combinatorial structures and the associated exponential generating functions. The passage from species to generating functions is a combinatorial analog of the Fourier transform. Indeed, there is a convolution multiplication on species that is mapped to a pointwise multiplication of the exponential generating functions. The statistical mechanics side deals with a probability model of an equilibrium gas. The cluster expansion that gives the density of the gas is the exponential generating function for the species of rooted connected graphs. The main results of the theory are simple criteria that guarantee the convergence of this expansion. It turns out that other problems in combinatorics and statistical mechanics can be translated to this gas setting. So, it is a universal prescription for dealing with systems of high dimension.

MSC:

82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics

Cited in **13** Documents

60K35 Interacting random processes; statistical mechanics type models; percolation theory

82B05 Classical equilibrium statistical mechanics (general)

05C30 Enumeration in graph theory

05A15 Exact enumeration problems, generating functions

82D60 Statistical mechanics of polymers

Keywords:

equilibrium lattice gas; polymer system; cluster expansion; species of structures; exponential generating function; connected graph

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