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**Efficiency of mixed hybrid finite element and multipoint flux approximation methods on quadrangular grids and highly anisotropic media.** (English) Zbl 1195.74208

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Summary: The mixed hybrid finite element (MHFE) and the multipoint flux approximation (MPFA) methods are well suited for anisotropic heterogeneous domains since both are locally conservative and can handle general irregular grids. In this work, behaviours and performances of MHFE and MPFA methods are studied numerically for different heterogeneities and anisotropy factors on parallelograms and then on a more general quadrilateral grid.

The superiority of MPFA in terms of accuracy and efficiency is clearly demonstrated for parallelogram grids. In the case of more general quadrilateral grids, MPFA becomes more central processing unit time consuming than MHFE. For high anisotropy factors, both methods give results with significant non-physical oscillations.

**MSC:**

- 74S05 Finite element methods applied to problems in solid mechanics
- 74S30 Other numerical methods in solid mechanics (MSC2010)
- 74F10 Fluid-solid interactions (including aero- and hydro-elasticity, porosity, etc.)
- 74E05 Inhomogeneity in solid mechanics

Cited in **13** Documents

**Keywords:**

mixed hybrid finite element; multipoint flux approximation; continuity point; high anisotropy; heterogeneous media; quadrilateral mesh

**Full Text:** [DOI](#)

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