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Description of B-orbit closures of order 2 in upper-triangular matrices. (English)

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Summary: Let $\mathfrak{n}_n(\mathbb{C})$ be the algebra of strictly upper-triangular $n \times n$ matrices and let $\mathcal{X}_2 = \{u \in \mathfrak{n}_n(\mathbb{C}) \mid u^2 = 0\}$ be the subset of matrices of nilpotent order 2. Let $\mathbf{B}_n(\mathbb{C})$ be the group of invertible upper-triangular matrices acting on $\mathfrak{n}_n$ by conjugation. Let $\mathcal{B}_u$ be the orbit of $u \in \mathcal{X}_2$ with respect to this action. Let $\mathbf{S}_n^2$ be the subset of involutions in the symmetric group $\mathbf{S}_n$. We define a new partial order on $\mathbf{S}_n^2$ which gives the combinatorial description of the closure of $\mathcal{B}_u$. We also construct an ideal $\mathcal{I}(\mathcal{B}_u) \subset S(\mathfrak{n}^*)$ whose variety $\mathcal{V}(\mathcal{I}(\mathcal{B}_u))$ equals $\overline{\mathcal{B}}_u$. We apply these results to orbital varieties of nilpotent order 2 in $\mathfrak{sl}_n(\mathbb{C})$ in order to give a complete combinatorial description of the closure of such an orbital variety in terms of Young tableaux. We also construct the ideal of definition of such an orbital variety up to taking the radical.

MSC:

14L30 Group actions on varieties or schemes (quotients)
17B08 Coadjoint orbits; nilpotent varieties
05E10 Combinatorial aspects of representation theory

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