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Weights of mixed tilting sheaves and geometric Ringel duality. (English) [Zbl 1197.14016]

Let $k = \mathbb{F}_q$, and let $X = \sqcup \alpha X_\alpha$ be a stratified scheme over $k$. Denote by $D^b_m(X)$ the bounded derived category of mixed complexes of $\mathbb{Q}_\ell$-sheaves on $X$ ($\ell$ prime to $q$). For each $\alpha$, let $\Delta_\alpha$ and $\nabla_\alpha$ be the standard and costandard complexes in $\text{Perv}_m(X)$. Under the condition $(\Diamond)$, that the integer-weight-twists of $\Delta_\alpha$ and $\nabla_\alpha$ generate the full triangulated subcategory of $D^b_m(X)$, the author considers mixed tilting ({$\ell$}-adic) sheaves on $X$. As an example, $(\Diamond)$ holds if the stratification of $X$ is given by orbits under an algebraic group action.) The author finds indecomposable mixed tilting extensions of $\mathbb{Q}_\ell \cdot X_\alpha$ $(\dim X_\alpha)$ (dim $X_\alpha/2$). Here, “indecomposable” means that the non-mixed complex, obtained by applying the forgetful functor into the bounded derived category of $\mathbb{Q}_\ell$-complexes with constructable cohomology on the geometric fiber of $X$, is indecomposable.

Let $T$ be an indecomposable tilting extension of $\mathbb{Q}_\ell \cdot X_\alpha$ $(\dim X_\alpha)$ . The focus of this paper is to compute the weights of $T$. If $T$ is Verdier self-dual then the coefficients of its weight polynomials $W_\alpha(T,t)$ satisfy a triangular system of linear equations which can be solved uniquely. Suppose now that $f : X = \sqcup \alpha X_\alpha \to Y = \sqcup \beta Y_\beta$ is a proper morphism of stratified schemes, each of which satisfy $(\Diamond)$, such that $f$ respects the stratification. Suppose furthermore that $f_\alpha : X_\alpha \to Y_{\phi(\alpha)}$ is a trivial fibration whose fibers are affine spaces. (Here $\phi$ is the identification between the indices on $X$ and $Y$ given by $f$.) Let $T$ be a mixed tilting extension so that for each $\beta < \alpha$ we have that $i_\beta^*T$ is of weight at least one, and $i_\beta^!T$ is of weight no larger than $-1$ (“the weight condition”). Then a formula for $W_\beta(f_!T_\alpha,t)$ is given, depending only on $W_\alpha(T_\alpha,t)$ for $\gamma \leq \beta \leq \alpha$. Finally, suppose there is a Radon transform $R_{X \to Y}$. Then the underlying non-mixed complex of $R_{X \to Y}(T)$ is a projective cover of an IC sheaf, and the weight polynomials are given in terms of the mixed stalks of the IC sheaves on $Y$.

As an application, if $X$ is an affine flag variety with Schubert stratifications then there is one indecomposable mixed tilting extension $T_\bar{w}$ of $\mathbb{Q}_\ell(\ell(\bar{w}))$ on $X_{\bar{w}}$, where $\bar{w} \in \bar{W}$ ($\bar{W}$ the affine Weyl group parameterizing certain orbits) The weight polynomials of $T_\bar{w}$ are given in a most explicit manner, depending on the Kazhdan-Lusztig polynomials for $\bar{W}$. Furthermore, $T_\bar{w}$ satisfies the weight condition above. A similar result holds for affine partial flag varieties.

Reviewer: Alan Koch (Decatur)

MSC:
14F20 Étale and other Grothendieck topologies and (co)homologies
14L30 Group actions on varieties or schemes (quotients)
20G25 Linear algebraic groups over local fields and their integers

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