Abraham, Uri; **Magidor, Menachem**

Cardinal arithmetic. (English) Zbl 1198.03053

Foreman, Matthew (ed.) et al., Handbook of set theory. In 3 volumes. Dordrecht: Springer (ISBN 978-1-4020-4843-2/hbk; 978-1-4020-5764-9/ebook). 1149-1227 (2010).

Summary: Following the work of Gödel and Cohen we now know that it is impossible to determine the exact value of the continuum or of the power set of an arbitrary cardinal. Despite the limitations that the consistency methods impose, the quest for absolute answers as to the value of the power set (especially of singular cardinals) continues, and sometimes with surprising and unexpected directions. The pcf theory developed by Shelah and considerably extending previous work of Silver and Galvin and Hajnal and others is an important avenue in investigating set-theoretical and combinatorial questions about the power-set of singular cardinals. The aim of our chapter is to give a self-contained development of this theory and related results as well as some of its interesting applications. For example the celebrated theorem of Shelah that gives an absolute estimate on the number of countable subsets of $\aleph_\omega$ (the first singular cardinal). The question whether this limitation can be improved is considered to be one of the main open problems in set theory.

See also the review of the complete volume [Zbl 1197.03001].

For the entire collection see [Zbl 1197.03001].

MSC:

03E10 Ordinal and cardinal numbers
03E04 Ordered sets and their cofinalities; pcf theory

Full Text: DOI