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Unstable blowups. (English) Zbl 1198.14046


Let \((X, L)\) be a polarized manifold of dimension \(n\), \(Z \subset X\) a subscheme and \(\nu : \text{Bl}_Z X \to X\) the blow up of \(X\) along \(Z\) with exceptional divisor \(E\). If \(X\) has a 1-parameter subgroup of automorphisms \(\alpha : \mathbb{C}^* \to \text{Aut}(X)\), then taking the limit of \(\alpha\) as \(t \to 0\) one obtains a test configuration \(X\) for \((\text{Bl}_Z X, \nu^* L - E)\) (note that the line bundle \(\nu^* L - E\) is ample for \(\gamma \gg 0\)). This configuration is intuitively given by “making the components of \(E\) move around and possibly collide”. One would like to make this construction rigorous and explicit so as to be able to compute the first terms of the asymptotic expansion of the Futaki invariant \(F(X)\) as \(\gamma \to \infty\).

In the paper under review, the author carries out this program for 0 dimensional cycles \(Z\). \(F(X)\) is then computed in terms of the classical Futaki invariant \(F(X)\) (for the holomorphic vector field generating \(\alpha\)) and of the natural GIT weight for the action of \(\alpha\) on the corresponding Chow variety of 0-cycles. As a corollary, it is shown that if \((X, L)\) is asymptotically Chow polystable, its blowup along a Chow unstable 0-cycle is asymptotically Chow unstable, when the exceptional divisors are small enough (i.e. \(\gamma \gg 0\)). New examples of Kähler classes with no constant scalar curvature representatives are given.

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MSC:

14L24 Geometric invariant theory
14E05 Rational and birational maps
14C20 Divisors, linear systems, invertible sheaves

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References:


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