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Entropy geometry and disjointness for zero-dimensional algebraic actions. (English)
Zbl 1198.37010

From the text: We discuss some mutual disjointness properties of algebraic actions of higher-rank abelian groups on zero-dimensional groups. The tools used are a version of the half-space entropies introduced by B. Kitchens and K. Schmidt [Ergodic Theory Dyn. Syst. 9, 691–735 (1989; Zbl 0709.54023)] and adapted by M. Einsiedler [Monatsh. Math. 144, No. 1, 39-69 (2005; Zbl 1061.37006)], a basic geometric entropy formula from that last cited paper, and the structure of expansive subdynamics for algebraic \( \mathbb{Z}^d \)-actions due to M. Einsiedler, D. Lind, R. Miles and T. Ward [Ergodic Theory Dyn. Syst. 21, 1695–1729 (2001; Zbl 1003.37003)]. We show that any collection of algebraic \( \mathbb{Z}^d \)-actions on zero-dimensional groups with entropy rank or co-rank one that look sufficiently different are mutually disjoint. The main results are the following (here \( N(\cdot) \) denotes the set of non-expansive directions defined in the paper).

Theorem 5.1. Let \( X_1, \ldots, X_n \) be a collection of irreducible algebraic zero-dimensional \( \mathbb{Z}^d \)-actions, all with entropy rank one. If

\[
N(\alpha_j) \cup_{k>j} N(\alpha_k) \neq \emptyset \quad \text{for} \quad j = 1, \ldots, n
\]

then the systems are mutually disjoint.

The simplest illustration of Theorem 5.1 is the fact that Ledrappier’s Example 2.3 and its mirror image are disjoint. This is shown directly in Section 3 to illustrate how the Abramov–Rokhlin formula for half-space entropies may be used.

Theorem 6.2. Let \( Y \) and \( Z \) be prime \( \mathbb{Z}^d \)-actions with entropy co-rank one. If \( N(\alpha_Y) \neq N(\alpha_Z) \), then \( Y \) and \( Z \) are disjoint.

Once again the simplest illustration of the meaning of this result comes from an example of Ledrappier type: Example 6.3 is a three-dimensional analogue of Ledrappier’s example. This is a \( \mathbb{Z}^3 \)-action defined by a ‘four-dot’ condition which has positive entropy \( \mathbb{Z}^2 \)-subactions; it and its mirror image are disjoint.

Surprisingly, it is not the familiar presence of different non-mixing sets but the entropy and subdynamical geometry of the systems that forces this high level of measurable difference of structure. The methods should extend to entropy rank or co-rank greater than one, but the notational and technical difficulties become more substantial.

**MSC:**

- 37A15 General groups of measure-preserving transformations and dynamical systems
- 28D20 Entropy and other invariants
- 37A35 Entropy and other invariants, isomorphism, classification in ergodic theory

**Keywords:**

mutual disjointness properties of algebraic actions of higher-rank abelian groups on zero-dimensional groups; Abramov–Rokhlin formula

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