Rémy, Bertrand; Thuillier, Amaury; Werner, Annette
Bruhat-Tits theory from Berkovich’s point of view. I: Realizations and compactifications of buildings. (English) [Zbl 1198.51006]

In this beautifully written article, the authors investigate Bruhat-Tits buildings and their compactifications by means of Berkovich analytic geometry over complete non-Archimedean local fields.

For a \( k \)-isotropic semisimple algebraic group \( G \) defined over a non-Archimedean valued field \( k \) and for the corresponding Euclidean building \( B(G, k) \) the authors prove that, if \( k \) is complete with respect to its valuation and if \( G \) is almost \( k \)-simple, then for any conjugacy class of proper parabolic \( k \)-subgroups, of type \( t \), there exists a continuous \( G(k) \)-equivariant map \( \theta_t : B(G, K) \rightarrow \text{Par}_t(G)^{\text{an}} \) which is a homeomorphism onto its image; the closure of this image is called the Berkovich compactification of type \( t \) of the given Bruhat-Tits building. Cf. [V. G. Berkovich, Spectral theory and analytic geometry over non-archimedean fields, Mathematical Surveys and Monographs, 33. Providence, RI: American Mathematical Society (AMS). (1990; Zbl 0715.14013)] for the notion of a Berkovich \( k \)-analytic space associated to a \( k \)-variety.

The map \( \theta_t \) in fact exists whenever the non-Archimedean valued field \( k \) is such that the Bruhat-Tits building \( B(G, k) \) exists functorially [cf. G. Rousseau, Publ. Math. D’Orsay 77-68, 207 p. (1977; Zbl 0412.22006)].

The paper heavily relies on an intimate knowledge of the theory of algebraic group schemes, and is a very welcome example of how to efficiently work with functoriality properties without losing oneself in abstract category theory.

Reviewer: Ralf Gramlich (Darmstadt)

MSC:

51E24 Buildings and the geometry of diagrams
20E42 Groups with a \( BN \)-pair; buildings
14L15 Group schemes

Keywords:

Bruhat-Tits building; algebraic group scheme; non-archimedean local field; compactification; Berkovich analytic space

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