This paper extends theorems of D. Montgomery [Bull. Am. Math. Soc. 42, 879–882 (1936; Zbl 0015.39403)] and R. Ellis [Proc. Am. Math. Soc. 8, 372–373 (1957; Zbl 0079.04104)] to a class of newly introduced fan-complete spaces. Among fan-complete spaces are the Čech-complete ones and here it is proven that in case of paracompactness every fan-complete space is Čech-complete, too. Another interesting property of fan-complete spaces is that, if a space is fan-complete locally, then it is so globally (Proposition 6). The main result is Theorem 7, stating that, if a paratopological group has a dense fan-complete subspace, then it must be in fact a topological group. An interesting interrelation between the existence of a dense fan-complete subspace $X$ in a topological group $G$ and the way how it is located in $\beta^*X$ is given in Theorem 10, stating that this happens if and only if $G$ is a $G_\delta$-subspace of $\beta^*X$. An open problem about Raikov complete groups is posed: If a Raikov complete group $H$ is a subgroup and a subspace of a quasitopological group $G$, is then $H$ closed in $G$? This reflects the authors’ result given in Corollary 11.

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MSC:

- 54H11 Topological groups (topological aspects)
- 54H20 Topological dynamics (MSC2010)
- 54H15 Transformation groups and semigroups (topological aspects)

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