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Construction of complete embedded self-similar surfaces under mean curvature flow. II.

(English) Zbl 1200.53061
Adv. Differ. Equ. 15, No. 5-6, 503-530 (2010).

The author studies surfaces in $\mathbb{R}^3$ which satisfy the equation

$$H + X \cdot \nu = 0,$$

where X, ν and H are the position-vector, the unit normal and the mean curvature respectively. Such surfaces are called shrinking self-similar surfaces or self-shrinkers because the mean curvature flow does not change their shape and merely contracts them.

The graph of a function u over a domain $\Omega \subset \mathbb{R}^2$ satisfies the self-shrinker equation if and only if u satisfies the equation

$$\mathcal{E}(u) = \left(\delta^{ij} - \frac{D_i u D_j u}{1 + |Du|^2}\right)(Du(\xi) D_{ij} u(\xi) - \xi \cdot Du(\xi) + u(\xi)) = 0,\ \xi \in \Omega.$$

The author discusses a Dirichlet problem for this equation on an unbounded domain. The main result states that, for small enough boundary conditions with some symmetries on the circle $C_R = \partial D_R$ of radius R in the plane, there exists a function u matching the boundary conditions such that the graph of u outside the disc D_R is a self-shrinker.

Theorem. Let $\sqrt{3}/2 < R < 2$ and $N \geq 5$. There is an $\varepsilon_0 > 0$ depending on R and N such that, for any $f \in C^4([0, 2\pi])$ with $||f||_{C^4([0,2\pi])} = \varepsilon \leq \varepsilon_0$ and satisfying the symmetries $f(\theta) = -f(-\theta) = f(\pi/N - \theta)$, there exists a function u on $\Omega = \mathbb{R}^2 \setminus D_R$ such that

$$\mathcal{E}(u) = 0 \quad \text{in } \Omega,$$

$$u = f \quad \text{on } \partial D_R,$$

$$u(r, \theta) = -u(r, -\theta) = u(r, \pi/N - \theta) \quad \text{for } r > R,\ \theta[0, 2\pi).$$

Moreover, we can choose the constant ε_0 uniformly for all $R \in (\sqrt{3}/2, 2)$.

Here r, θ are the polar coordinates in $\mathbb{R}^2$ whose pole coincides with the center of D_R, so $u = f$ on ∂D_R means that $u(R, \theta) = f(\theta)$.

In the proof the solution to $\mathcal{E}(u) = 0$ is found considering the limit for time t going to infinity of a solution to the parabolic equation $\partial_t u = \mathcal{E}(u)$.

The author suggests that the result obtained may be applied to construct new examples of complete embedded self-similar surfaces under mean curvature flow [cf. *N. Kapouleas*, J. Differ. Geom. 47, No. 1, 95–169 (1997; Zbl 0936.53006) and the first part, *X. H. Nguyen*, Trans. Am. Math. Soc. 361, No. 4, 1683–1701 (2009; Zbl 1166.53046)].

Reviewer: Vasyl Gorkaviy (Kharkov)

MSC:

53C44 Geometric evolution equations (mean curvature flow, Ricci flow, etc.) (MSC2010)

Keywords:

self-shrinkers; Dirichlet problem

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