Charalambides, Marios; Waleffe, Fabian
Gegenbauer tau methods with and without spurious eigenvalues. (English) Zbl 1201.65133

The paper is concerned with the numerical solution to a fourth order differential eigenvalue problem

$$D^4 u = \lambda D^2 u \quad \text{in} \quad -1 \leq x \leq 1; \quad u = Du = 0 \text{ at } x = \pm1.$$  \hspace{1cm} (1)

Various existing methods for problem (1) are investigated in the context of Gegenbauer methods with residuals weighted by $W(\gamma) = (1-x^2)^{\gamma-1/2}$, where $\gamma = 0$ corresponds to Chebyshev and $\gamma = 1/2$ to Legendre polynomials. It is proven that the Gegenbauer tau method applied to (1) provides eigenvalues that are real, negative, and distinct as in the case of the exact solutions when $1/2 < \gamma \leq 7/2$. Numerical calculation confirms that this range for $\gamma$ is sharp. Spurious positive eigenvalues exist for $\gamma < 1/2$ and complex eigenvalues arise for $\gamma > 7/2$. Furthermore, it is shown that the widely used modified tau method is mathematically equivalent to the Galerkin method.

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MSC:

65L15 Numerical solution of eigenvalue problems involving ordinary differential equations
34L16 Numerical approximation of eigenvalues and of other parts of the spectrum of ordinary differential operators
65L60 Finite element, Rayleigh-Ritz, Galerkin and collocation methods for ordinary differential equations

Keywords:
Gegenbauer tau methods; spurious eigenvalues; numerical examples; fourth order differential eigenvalue problem; positive eigenvalues; complex eigenvalues; Galerkin method

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