

[Brugallé, Erwan](#)

**Some tropical geometry. (Un peu de géométrie tropicale.)** (French) Zbl 1202.14055  
[Quadrature 74, 10-22 \(2009\)](#).

This paper is an interesting and enjoyable light introduction to tropical geometry. It contains the statements of several theorems, but only a few easy proofs and many examples. The author explains the basic facts of tropical algebra: addition, multiplication, polynomials, etc. Then he tells us about tropical lines, conics and more general tropical algebraic curves. Key concepts such as the dual subdivision, the balance condition at vertices and multiplicities are explained in simple words. Tropical algebraic curves satisfy Bézout's theorem. In fact, tropical geometry is a simplified (piecewise linear, combinatorial) version (or shadow) of complex classical algebraic geometry. This relationship provides a new way to prove complex algebraic geometry theorems. This makes tropical geometry interesting to many mathematicians, including the author.

Let me recall that Hilbert's 16th problem asks, for any degree  $d$ , for the possible arrangements of the connected components of a real plane algebraic curve of degree  $d$ . This problem remains open. A fruitful technique to address this hard problem, called patchwork, was developed in the 70's. In the 90's a process called dequantization of real numbers was introduced. It allows us to interpret tropical sum and product over the reals as a limit (as  $t$  tends to infinity) of classical sum and product, composed with base  $t$  logarithm. And patchwork can be interpreted as a quantification (the reverse process) of tropical curves. This is all explained in the paper.

A nice set of exercises (with answers available on line) helps the interested reader to try his/her way through the (yet in many ways unexplored) tropical mathematical world.

Reviewer: [María Jesús de la Puente \(Madrid\)](#)

**MSC:**

[14T05](#) Tropical geometry (MSC2010)  
[14P05](#) Real algebraic sets  
[14N05](#) Projective techniques in algebraic geometry

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