Durrett, Rick

Probability. Theory and examples. 4th ed. (English) Zbl 1202.60001

Cambridge Series in Statistical and Probabilistic Mathematics 31. Cambridge: Cambridge University Press (ISBN 978-0-521-76539-8/hbk). x, 428 p. (2010).

This is the 4th edition of this text book on probability theory, which has become a classic by now. For the review of the 1st edition, cf. [The Wadsworth & Brooks/Cole Statistics/Probability Series. Pacific Grove, CA: Wadsworth & Brooks/Cole Advanced Books & Software. (1991; Zbl 0709.60002)]. Compared to the 3rd edition, some parts of the appendix have been combined with parts of Chapter 1, to form a new first chapter on measure theory, thus not requiring any previous knowledge of measure theory any more.

The text can be used as the basis for a one- or two-semester course in probability theory. It treats the basics of probability theory such as the law of large numbers and the central limit theorems, but also discrete time stochastic processes and ergodic theory. It ends with an introduction to Brownian motion. As the title indicates, a strong focus is on examples (there are roughly 200 examples in the text). Also, there is a great selection of exercises (450 of them), because as the author states in the preface "probability is not a spectator sport".

The first chapter is a short and concise introduction to measure theory, which should help students without any previous knowledge of measure theory to understand the following chapters. Some of the more complicated results are not included here but still rather found in the appendix.

The second chapter treats laws of large numbers in some detail: The concept of independence is introduced and then applied to give weak and strong laws of large numbers. The Borel-Cantelli lemmas are also proven here. There even is a section on large deviations.

The next chapter focuses on central limit theorems. The concepts of weak convergence and of characteristic functions are introduced and as everywhere else in the text, the formal definitions and results are accompanied by a multitude of examples and exercises to help the reader really grasp these concepts. This is then applied to prove central and local limit theorems. At the end of the chapter there are sections on stable laws and infinitely divisible distributions.

Then there is a chapter on random walks. Stopping times are introduced and used to prove various results about random walks. There is a section on recurrence of general random walks which uses mostly analytic methods, and one on recurrence results for the symmetric random walk on Z, which uses mostly combinatorial methods.

The 5th chapter gives the basic theory of discrete times martingales, such as convergence theorems or Doob's inequality.

The next chapter does the same for discrete time Markov processes in countable state spaces. The focus is on recurrence and transience results. At the end of the chapter, there is a short section on Markov chains in general state spaces.

The 7th chapter is on ergodic theorems. Birkhoff's ergodic theorem is presented, but there is also a section on subadditive ergodic theorems.

The 8th and last chapter is on Brownian motion. It is constructed using Kolmogorov's continuity criterion, and some of its properties such as the Markov property or path properties are presented. Donsker's theorem is also stated. There even is a section on empirical distributions and their relation to the Brownian bridge, which is often omitted in introductory probability courses.

The book finishes with an appendix on some deeper results from measure theory,

This text book has probably served thousands of students over the past 20 years as an introduction to probability theory, and the 4th edition will certainly keep doing so.

Reviewer: Nicolas Perkowski (Berlin)

MSC:

60-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to probability theory
60A05 Axioms; other general questions in probability
97K50 Probability theory (educational aspects)

Full Text: DOI