Sondow, Jonathan

A hypergeometric approach, via linear forms involving logarithms, to criteria for irrationality of Euler's constant. With an appendix by Sergey Zlobin. (English) Zbl 1203.11053

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In a previous paper [Proc. Am. Math. Soc. 131, No. 11, 3335–3344 (2003; Zbl 1113.11040)] the author, inspired by the work of Beukers on the irrationality of $\zeta(2)$ and $\zeta(3)$, considered the following sequence of double integrals

$$I_n = \int_0^1 \int_0^1 \frac{[x(1-x)y(1-y)]^n}{(1-xy)(-\log xy)}\, dx\, dy$$

and proved that $\left[[I_n - \binom{2n}{n}\gamma]\mathrm{lcm}(1,2,\dots,2n) \in Z + Z\log(n+1) + \cdots + Z\log(2n)\right]$. He then used this property to derive interesting criteria for the irrationality of Euler's constant γ.

In this paper he continues these investigations and obtains the equivalent single integral

$$I_n := \int_{n+1}^{\infty} \frac{(n!)^2\Gamma(t)}{(2n+1)\Gamma(2n+1+t)}\, {}_3F_2\left(\left. \begin{matrix} n+1, n+1, 2n+1 \\ 2n+2, 2n+1+t \end{matrix} \right| 1 \right) dt,$$

by first showing that this single integral is equal to a Nesterenko-type series, which when replaced in the expression one obtains the same linear combination. In an appendix, S. Zlobin proves again, but without expanding in linear forms, that the double integral equals that Nesterenko-type series. Sondow hopes that the variety of expressions for I_n will turn out to be useful in determining the arithmetic nature of γ.

Reviewer: Jesús Guillera (Zaragoza)

MSC:

11J72 Irrationality; linear independence over a field
11J86 Linear forms in logarithms; Baker's method
33C20 Generalized hypergeometric series, ${}_pF_q$

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Euler's constant; irrationality; hypergeometric; linear forms in logarithms

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References: