

**Sondow, Jonathan**

**A hypergeometric approach, via linear forms involving logarithms, to criteria for irrationality of Euler's constant. With an appendix by Sergey Zlobin.** (English) [Zbl 1203.11053](#)  
Math. Slovaca 59, No. 3, 307-314 (2009).

In a previous paper [Proc. Am. Math. Soc. 131, No. 11, 3335–3344 (2003; [Zbl 1113.11040](#))] the author, inspired by the work of Beukers on the irrationality of  $\zeta(2)$  and  $\zeta(3)$ , considered the following sequence of double integrals

$$I_n = \int_0^1 \int_0^1 \frac{[x(1-x)y(1-y)]^n}{(1-xy)(-\log xy)} dx dy$$

and proved that  $[[I_n - \binom{2n}{n}\gamma]\text{lcm}(1, 2, \dots, 2n) \in Z + Z \log(n+1) + \dots + Z \log(2n)]$ . He then used this property to derive interesting criteria for the irrationality of Euler's constant  $\gamma$ .

In this paper he continues these investigations and obtains the equivalent single integral

$$I_n := \int_{n+1}^{\infty} \frac{(n!)^2 \Gamma(t)}{(2n+1)\Gamma(2n+1+t)} {}_3F_2 \left( \begin{matrix} n+1, n+1, 2n+1 \\ 2n+2, 2n+1+t \end{matrix} \middle| 1 \right) dt,$$

by first showing that this single integral is equal to a Nesterenko-type series, which when replaced in the expression one obtains the same linear combination. In an appendix, S. Zlobin proves again, but without expanding in linear forms, that the double integral equals that Nesterenko-type series. Sondow hopes that the variety of expressions for  $I_n$  will turn out to be useful in determining the arithmetic nature of  $\gamma$ .

Reviewer: Jesús Guillera (Zaragoza)

**MSC:**

[11J72](#) Irrationality; linear independence over a field  
[11J86](#) Linear forms in logarithms; Baker's method  
[33C20](#) Generalized hypergeometric series,  ${}_pF_q$

Cited in 1 Document

**Keywords:**

[Euler's constant](#); [irrationality](#); [hypergeometric](#); [linear forms in logarithms](#)

**Full Text:** [DOI](#)

**References:**

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.