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Hypergeometric transformations of linear forms in one logarithm. (English) Zbl 1205.33008

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The authors discuss constructions of rational approximations to values of the logarithmic function based on the hypergeometric series

$$H(a,b,c;\lambda)=\frac{1}{\lambda^{2c+2}}\frac{\Gamma(a+1/2)b!}{\Gamma(a+b+3/2)}{}_2F_1(c+1,a+1/2;a+b+3/2;1/\lambda^2),$$

where a, $b\geq 0$, $c\geq 0$ are integers and λ is an algebraic number with $|\lambda|>1$ and $\lambda\in K$ (examples include the fields $K=\mathbb{Q}$ and $K=\mathbb{Q}(\sqrt{D})$, where $D>1$ is a square-free even integer), and variants obtained by hypergeometric transformations (including the special case $c=a$). The series approach allows $a<0$ whereas the method using the integral form (utilized in [*E. S. Sal'nikova*, Math. Notes 83, No. 3, 389–398 (2008; Zbl 1201.11073)], also *V. Kh. Salikhov* [Dokl. Math. 76, No. 3, 955–957 (2007; Zbl 1169.11032)]) works for non-negative integers a, b, c. Although, the structure of the obtained rational approximations does not allow the authors (as remarked in the paper) to obtain improvements on the known irrationality results or irrationality measures, it leads to several remarks and two questions: one concerns a unified approach to the studied approximations and the other concerns the curious phenomenon that for specific choices of parameters in $H(a,b,c;\lambda)$ the corresponding linear approximations may have rational coefficients even if λ is a quadratic irrationality.

Reviewer: Johann Brauchart (Graz)

MSC:

33C05 Classical hypergeometric functions, ${}_2F_1$
11J82 Measures of irrationality and of transcendence
33C60 Hypergeometric integrals and functions defined by them (E, G, H and I functions)

Keywords:

Rational approximation; irrationality measure; hypergeometric series; hypergeometric integrals

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