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Compactness properties for some hyperspaces and function spaces. (English) Zbl 1206.54020

Berichte aus der Mathematik. Aachen: Shaker Verlag; Rostock: Univ. Rostock, Mathematisch-Naturwissenschaftliche Fakultät (Diss.) (ISBN 3-8322-0771-6). 91 p. (2002).

Summary: "Compactness properties of function space topologies, usually referred to as Ascoli-Arzela type results, are most fundamental and useful in both Topology and Analysis." (S. A. Naimpally) Here we contribute new aspects and strong new results to this widely studied topic through an unusual approach involving hyperspaces. We consider topological spaces and set-open topologies as well as we study a generalization of Tukey's approach to uniformity, namely the strong topological universe of multifilter spaces and fine maps, which may be viewed as the extension of the classical (and not unsubstantiated) dichotomy in descriptions of uniform structures into the realm of 'convenient topology', developed by Gerhard Preuß. Hyperspaces are studied for topological spaces as well as for multifilter spaces. Mostly emphasized are compactness properties for hit-and-miss topologies from topological spaces, simply, because they form the model for our new approach to Ascoli-theorems in this work.

Nevertheless, not all results are completely devoted to this attempt – we think, some could be interesting in their own right. There is a fairly useful set-theoretical lemma, for instance, and a property called 'weak relative complete' is considered for subsets of topological spaces. It is a common generalization of closedness and compactness, and in fact it is exactly what is needed to get compactness from relative compactness. It is proved that a hit-and-miss hyperspace, containing at least the nonempty closed subsets, is compact if and only if the base space is, whenever the miss-sets come from weak relative complete subsets. Most of the former known compactness results for Fell or Vietoris topology follow easily from this. Furthermore, a few results on (relative) compactness of unions of (relative) compact subsets are established. Concerning hyperstructures from multifilter-spaces, we feel a quite direct transcription of the Vietoris-construction being fruitful and we give a lemma concerning precompactness of unions of precompact sets here. Finally, we devote the main part of the text to the study of the simple but important map, as provided in a recent (1998) embedding theorem of Mizokami. The topological behaviour of this map is the key tool, that allow us to use our knowledge on hyperspaces to produce powerful theorems of the Ascoli-Arzela type.

MSC:

54D30 Compactness
54B20 Hyperspaces in general topology
54C35 Function spaces in general topology
46A50 Compactness in topological linear spaces; angelic spaces, etc.
46B50 Compactness in Banach (or normed) spaces