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Smoothness and jet schemes. (English) Zbl 1207.14023

Brasselet, Jean-Paul (ed.) et al., Singularities, Niigata-Toyama 2007. Proceedings of the 4th Franco-Japanese symposium, Niigata, Toyama, Japan, August 27–31, 2007. Tokyo: Mathematical Society of Japan (ISBN 978-4-931469-55-6/hbk). Advanced Studies in Pure Mathematics 56, 187-199 (2009).

Jet schemes over a variety encode information about its singularities. Given a smooth variety X , it is a well-known fact that the jet-schemes X_m are non-singular for every $m \in \mathbb{N}$ and that every truncation morphism $X_{m'} \rightarrow X_m$, for $m' > m$, is smooth. In this article, the author answers the question, whether these properties characterize smoothness of X , and obtains results which are even a bit stronger:

A morphism of k -schemes $f : X \rightarrow Y$ is smooth if and only if there is one $m \in \mathbb{N}$ such that $f_m : X_m \rightarrow Y_m$ is smooth. Hence a scheme X of finite type over k is smooth if and only if one X_m is smooth. Using truncation morphisms, a similar criterion is formulated for an algebraically closed field k of characteristic zero: A scheme of finite type over k is non-singular iff one truncation morphism $X_{m'} \rightarrow X_m$ is a flat morphism. In positive characteristic, this criterion does not hold as is illustrated in an example; but with the additional condition that X is reduced, an analogous statement is proved.

For the entire collection see [Zbl 1181.00034].

Reviewer: Anne Frühbis-Krüger (Hannover)

MSC:

14E15 Global theory and resolution of singularities (algebraic-geometric aspects)

14J17 Singularities of surfaces or higher-dimensional varieties

Cited in **3** Documents

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arc spaces; jet schemes; smoothness

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