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**Incompressibility of germs of singular holomorphic foliation. (Incompressibilité des feuilles de germes de feuilletages holomorphes singuliers.)** (French) Zbl 1207.32028

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This paper is dedicated to the study of local topological properties of (leaves of) germs of holomorphic foliations with an isolated singularity in complex dimension two. Two main hypotheses are made for the foliation: 1. The foliation is non-dicritic, which means that there are only finitely many separatrices, i.e., analytic invariant curves passing through the singularity. This is also equivalent to the fact that the resolution process by the blow-up method for the singularity of the foliation, originates only invariant projective lines. 2. The foliation is generic. This more vague notion is related to the type singularities arising in the resolution process. Thus they exclude the saddle-node case and assume that irrational singularities (i.e., those having irrational quotient of eigenvalues) are analytically linearizable.

For this class of germs of foliations the authors prove their main result which, in few words, assures that given such a foliation  $\mathcal{F}$ , defined in a ball  $B$  containing the origin, which is the singularity and with total set of separatrices  $S$ , there exists an open subset  $U \supset S$  of  $B$ , such that for every leaf  $L$  of  $\mathcal{F}|_{(U \setminus S)}$  the natural inclusion  $\iota : L \hookrightarrow U \setminus S$  induces a monomorphism  $\iota_* : \pi_1(L) \hookrightarrow \pi_1(U \setminus S)$  for the fundamental groups.

In order to achieve this, the authors introduce several notions among them notion of *foliated connexity*, which gives a re-interpretation of incompressibility. Using then the existence of suitable transverse holomorphic sections, constructed from the hypothesis on the foliation, the authors are able to give a *global monodromy representation* for the foliation. The paper is long and very technical. Despite the efforts of the authors the paper is not so easy to follow and ends to be intended for specialists or to people who are acquainted with the techniques of holomorphic foliations with singularities. Special attention is given to the notion of *rugosity* which is quite interesting and is a more geometrical ingredient in the algebraic context of the proof. The relatively short list of references indicates the fact that the paper is intended to be a treaty on the subject, which may be useful but also puzzling. A shortened version of the proof of the main result should be welcome.

Reviewer: [Bruno Scardua \(Rio de Janeiro\)](#)

**MSC:**

[32S65](#) Singularities of holomorphic vector fields and foliations

[37F75](#) Dynamical aspects of holomorphic foliations and vector fields

Cited in **6** Documents

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