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An existence result for a quadrature surface free boundary problem. (English) Zbl 1207.35106
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Summary: The aim of this paper is to present two different approaches in order to obtain an existence result to the so-called quadrature surface free boundary problem. The first one requires the shape derivative calculus while the second one depends strongly on the compatibility condition of the Neumann problem. A necessary and sufficient condition of existences is given in the radial case.

MSC:

35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation Cited in 3 Documents
35A35 Theoretical approximation in context of PDEs

Keywords:

Dirichlet problem; Neumann problem; quadrature surface; maximum principle, shape optimization; optimality conditions

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